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AN APPLICATION OF DECOMPOSITION TECHNIQUES TO A DECENTRALIZED FORCE-LEVEL DECISION PROBLEM

by
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Title of Thesis: An Application of Decomposition Techniques to a Decentralized Force-Level Decision Problem

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ABSTRACT

Thesis Title: An Application of Decomposition Techniques to a
Decentralized Force-Level Decision Problem

William E. Hutchison, Master of Arts, 1967.

Thesis directed by: Edward S. Pearsall, Ph.D.
Paul Wonnacott, Ph.D.

An approach to a force level problem which incorporates inputs of cost, operational effectiveness and requirements into a decision procedure is suggested. The problem is then defined within the framework of a decomposable linear program. The linear program is given an interpretation which emphasizes operational effectiveness at the point of use. A central authority -- subordinate element dialogue is postulated in the interpretation and conducted in the presence of the market mechanism implied by the decomposable linear program.

A specific task, illustrative of the more general technique, is taken as the derivation of a model which permits a central defense planner to resolve conflicts between military operational theaters for the purpose of programming construction of an amphibious force for use in all theaters.

A decentralized decision process for determination of force levels is described. The application of the decomposition algorithm to the central defense planner-military theater commander interchange employed in the decision process is developed.

The procedure under which amphibious force acquisition is presently accomplished is examined by solution of the linear program using a set of postulated theater tactical plans.

Conclusions are drawn which contrast the interpretation ordinarily given the decomposable linear program with the interpretation suggested for the model constructed. The general applicability of the model is discussed, means of improving and expanding the model presented and additional uses described.

AN APPLICATION OF DECOMPOSITION TECHNIQUES TO A DECENTRALIZED FORCE-LEVEL DECISION PROBLEM

by

William Edwin Hutchison

Thesis submitted to the Faculty of the Graduate School
of the University of Maryland in partial fulfillment
of the requirements for the degree of
Master of Arts
1967

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ACKNOWLEDGEMENT

The formulation and interpretation of the decomposable linear program used in this paper are based on a suggestion by Dr. Edward S. Pearsall, who subsequently devoted many patient hours to its development. Dr. Pearsall also programmed the model for computer solution of the force level problem.

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INTRODUCTION

The capability of the U.S. to protect its commitments abroad depends greatly on the availability of military forces trained and equipped for amphibious operations. The level of this military capability is restricted by the defense budget, however, and must compete with other equally critical defense requirements for funding. The Marine Corps and Navy face the problem of deciding upon the composition of a least cost amphibious force effective for operations in world-wide coastal maritime areas.

In particular, minimum cost force level objectives must be determined for an array of feasible types of amphibious ships and vehicles to accomplish a set of missions related to different geographical areas.

The difficulty is that contingency plans in likely theaters of operation impose conflicting requirements. Different types of military units are needed for amphibious operations in the Middle East (desert) and Southeast Asia (jungle). Heavily armored and highly ground-mobile forces are most effective in desert warfare. In contrast, armor is severely restricted in the jungle and lighter helicopterborne forces are more useful. Even more significant are the variations in expected opposition and sea and terrain conditions that would influence the conduct of amphibious operations in the separate theaters. Despite these differences in types of units and conditions of employment, budget considerations place a finite limit on the construction of forces of ships and vehicles for amphibious operations. Any force selected should clearly be capable of assault operations in all operational theaters, but should be acquired at minimum cost as well.

Allocation of Public Services - An Analogy

It may be useful to place this type of military force level problem in a more familiar context. The choice of an amphibious force which will operate effectively to meet conflicting requirements in different theaters finds an analogy in the more commonly encountered problem of furnishing public services. Consider, for example, the provision of the service of fire protection by a town.

The problem of providing fire protection as viewed by a local government can be visualized by assuming a setting in which a town is divided into a business section, a pier section, a housing section and an agricultural section. Suppose that the town is attempting to decide on the composition of a fire-fighting force which will provide fire protection to each section. Suppose also that the town budget will permit the purchase of only one set of equipment to be centrally positioned in the local firehouse. The problem is complicated by the characteristics of the separate sections, which are markedly dissimilar. The types of equipment most effective in one section may either be less useful or of no use at all in another. The relative value of a fireboat in the pier section, for example, would presumably differ from its value in the housing section.

The problem of selecting the most suitable composition of the fire-fighting force to be acquired can be approached by: (1) establishing the level of fire protection to be afforded to each section; (2) measuring the value of this level of protection in terms of potential property saved from damage in each section; and (3) determining the value of the

separate contribution to protection which different types of fire-fighting equipment can make to each section. If the sum of the per unit value of protection provided by a particular type of equipment over all sections of the town is less than its cost, it will not be purchased. Similarly if, at any point, the sum of the values attributed to an equipment type in the four sections exceeds its cost, additional units should be acquired up to the point where its value at the margin equals its cost.

The most difficult aspect of the approach suggested is the evaluation of the effectiveness of various types of equipment in the different sections. An alternative available to the town is to decentralize the decision by permitting each section to determine its own relative values for the different types of equipment. Although conflicts could be expected (because each section would attempt to maximize its own protection without regard to the impact of such behavior on the effectiveness of the fire-fighting force in other sections), participation of the sections would insure that the force selected is adequate to provide the established level of protection in every section.

An analytic technique which would allow each section of the town to participate in the selection of the fire-fighting force and at the same time coordinate the equipment evaluation procedure in a way which insures that the sum of the per unit values of each type of equipment acquired equals its cost would clearly be useful.

The explanatory value of the analogy between the problem of providing a given level of fire protection to separate sections of a town and that of selecting an amphibious force capable of meeting a stipulated

operational requirement is perhaps more apparent at this point. The fire-fighting force is mobile and can afford a certain level of fire protection to any section, using all or a portion of the available equipment. The amphibious force is also mobile and similarly can be deployed as a whole or in part to supply a given level of amphibious capability to any theater.[1] Also, one type of amphibious ship or vehicle may yield a very different effectiveness in each theater (as in the fire boat example of the town).

Another important similarity between the two problems is that the sum of the per unit values of the amphibious "protection" provided by a particular ship or vehicle should not exceed the cost of its acquisition.

A third parallel is observed in the expected reactions of the sections of the town and the theaters to the invitation to participate in the decision-making process. The theaters equally with the sections will attempt to influence the composition of the force in such a way as to insure optimal coverage of their own unique situations by attributing higher value to equipment most useful locally. The upshot is that, from an analytic viewpoint, the two problems are essentially the same.

Development of the Analogy

In the present paper, the analogy between the problem of furnishing fire protection facing the town and that of the central defense planner

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[1] An assumption is implied that not more than one fire (or contingency) will occur at a time. This assumption could obviously break down in either case, and is used only to limit the size of the requirement.

concerned with selecting a balanced military force at minimum cost is demonstrated by the development of a method of force level planning which relies on techniques of economic analysis applicable to both the fire-fighting and amphibious forces. The specific task, illustrative of a more general technique, is taken to be the derivation of a model which will enable the central planner to resolve conflicts in requirements between theaters for the purpose of programming construction of a force of amphibious vehicles and ships which can effectively transport and deliver a Marine landing force from ship to shore under varying conditions of geography, enemy opposition and landing force composition.

The force of amphibious ships and vehicles (called the amphibious force when combined) is akin to the fire-fighting force of a town in the sense that it must meet the demands for protection of its separate sections (theater contingency plans in the analogy) at minimum cost.

The analogy between the fire-fighting force decisions of the city government and defense force level decisions can be extended by supposing that commanders in the different operational theaters (sections of the town) have developed separate tactical plans which represent the most probable way in which an amphibious assault would be conducted in the particular area for which the commander is responsible. It would clearly be desirable to decentralize the decision-making process to permit the responsible commanders to cooperate by employing these plans as inputs to the analysis. Participation of the theater commanders in this manner should insure adequate consideration of military operational factors in the development of final amphibious force objectives.

In succeeding chapters, the problem of the central planner is given explicit form, and a technique for solving the problem through a decentralized decision process is developed. The procedure under which amphibious force levels are presently determined is also described and examined. Finally, the decision process suggested is employed to solve a specific force level problem, the solution is presented and conclusions are drawn.

CHAPTER I

DEFINITION OF THE PROBLEM AND CONSTRUCTION OF THE MODEL

Background

The Marine Corps is the responsible service within the Department of Defense for the development of systems and doctrines peculiar to amphibious operations. The present concept for development of future U.S. amphibious capabilities is based on a general tactical plan for amphibious operations judged by the Marine Corps to be effective for any geographical area and to provide an adequate forecast of force level requirements. The postulated concept is (in its effect on force objectives) a single plan which allocates certain landing force task organizations in terms of troops, equipment and supplies to objectives located ashore, to be delivered from Naval amphibious ships under specified parametric restrictions of time and distance by a particular means: helicopter, armored amphibians,or general amphibious vehicles. Planning for acquisition of the amphibious force is accomplished under the assumption that one plan (expressed as a set of operational parameters) is a suitable framework for selection of component ships and vehicles from a variety of feasible types in amounts sufficient to be effective over probable contingencies.

Although the parameters established by the Marine Corps as minimum amphibious force capabilities to be achieved are classified in part, an unclassified version of the general objective is available and is excerpted below:

"Utilization of the various tactical mobility means in proportions dictated by the requirements of the mission will provide Marine air-ground task forces with a capability to conduct initial assault operations at ranges up to 50 nautical miles inland, and over an area which, in the case of a force of division-wing size, may extend to approximately 600 square miles (roughly 20 by 30 miles)."[1]

The underlined sentence states that the "tactical mobility means available" (amphibious ships and vehicles) are to be employed in different proportions according to the mission. The amphibious force must operate effectively under fluctuating conditions of employment, but, in contrast, force level objectives are derived from one hypothetical plan. The point is that unless the plan provides more than enough amphibious ships and vehicles for every contingency, it is unlikely to result in a force which is most effective over a range of predictable missions. The present concept for resource acquisition is equally dubious from a cost viewpoint. If the plan results in excess ships and vehicles, then the cost of the force is greater than its value in the defense sector and other, more critical, activities in the government or private sector must be needlessly cut back. Potential deficiencies of the "one plan" technique will be demonstrated.

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[1] Employment of Marine Air-Ground Task Force in Future Amphibious Operations. Marine Corps Order 3340.3, 20 April 1962.

Definition of the Problem

Suppose that the Marine Corps has been requested by the Secretary of Defense to recommend a minimum cost amphibious force to meet approved contingency plans of unified (theater) commanders for the period 1970-1979.[1] Suppose also that the Secretary's request includes a restriction that the amphibious force recommended need be capable only of responding to one theater plan at a time, under the assumption that no two contingencies will arise simultaneously.[2] In order to act on the request, the Marine Corps designates a central planner who reviews the request and defines the problem: Develop recommendations for an optimal force of amphibious vehicles and ships for world-wide employment which takes explicit account at minimum cost of the differing operational requirements of the theater commanders.[3]

---

[1] The amphibious force is taken to include only the amphibious vehicles and the ships in which they are embarked to support the movement ashore. Other Navy and Marine Corps resources are not considered.

[2] This assumption could easily be relaxed by expanding the requirement to include development of a separate amphibious force for each contingency. It is not considered realistic, however, to expect that every contingency can or should be covered by a uniquely structured amphibious force. Nevertheless, the decision process suggested in the present paper can absorb any combination of amphibious forces and theaters preferred to that postulated.

[3] The definition of the problem may appear to beg the real question of optimality in the sense that it does not require analysis of alternatives to requirements (in magnitude) of theater contingency plans for subsequent policy level decision. This is true only to the extent that responsible commanders may overestimate the size of the military force needed to meet particular contingencies. The assumption implied by the definition is that theater requirements have previously been evaluated by the Secretary of Defense and found to be valid.

The Approach. Certain information is given the planner. Other information must be obtained. The planner determines an approach which will identify the basic elements of the problem and then apply these elements in an analytical model. The following procedure is settled upon:

1. Obtain theater plans assigning output missions (operational requirements) to the amphibious force which are defined by the tonnages of Marine landing force units to be delivered ashore in the (a) helicopter-borne assault, (b) armored amphibian assault, and by (c) general amphibious vehicle means.
2. Select an array of feasible candidate amphibious vehicles and ships expected to be available in 1970.
3. Determine two types of ship and vehicle costs:
    a. Investment and discounted peacetime operating costs
    b. Expected attrition costs of executing an amphibious operation in each theater.
4. Compute the individual output capacities of each ship and vehicle in the selected array in each theater.
5. Develop a model which minimizes cost subject to constraints which insure that the aggregate output of the selected force is sufficient to meet theater output missions.

6. Exercise the model using requirement, cost, and output data obtained or computed to determine an optimal "mix" of ships and vehicles and test the results.

Theater Output Missions. Theater tactical plans are submitted to the central planner by the appropriate commanders. Requirements are amphibious force output missions derived from these plans, which each commander considers representative of the nature of probable large-scale amphibious operations in his theater. The plans reflect differing conditions of expected enemy opposition, geographical environment and mission and composition of the Marine landing force. Each plan is expressed by output parameters:

1. Theater vehicle output missions (in tons) in three categories:
    a. Landing force maneuver elements to be transported by helicopter for the purpose of seizing inland objectives.
    b. Landing force maneuver elements to be transported in armored amphibians for the surface assault against objectives in the beach area.
    c. Combat support units of the landing force to be delivered by other general amphibious vehicle means.
2. The distance in miles from the amphibious ships to inland objectives divided into land and water segments.

3. The time in hours during which the initial assault is to take place.

Development procedures and rationale for differing tactical plans in four theaters are contained in Appendix 1. The amphibious force decided upon must be capable of performing the output missions generated by each of these plans. The theaters are Southeast Asia (SEASIA), Middle East (MIDEAST), Northern Europe (NOREUR) and one worldwide "theater", Counterinsurgency (CI). Operational conditions imposed by these theaters encompass the majority of situations in which amphibious operations are likely to be conducted in the period 1970-79.

Candidate Vehicles and Ships. Vehicle types and ship classes selected as feasible candidates by the central planner which are available at present or can be available in quantity by 1970 are listed and a brief description of the physical characteristics and output mission in which each operates is provided in Appendix 2. It is from this list that program objectives for the force "mix" for the period 1970-79 are to be set. There are at least two competitors for selection in each of the three output missions.[1]

Costs. The procedure for developing cost information used by the central planner places emphasis on two types of costs: investment plus peacetime operating costs (the latter discounted at 10% from the year 1970), and expected attrition costs. Investment costs for the

---

[1] No distinction is made as to which Service, Navy or Marine Corps, "owns" the vehicles.

year 1970 are assumed to equal current acquisition costs of the candidate vehicles and ships. Expected attrition costs for each theater are computed as the product of: 1) estimated vehicle attrition (which varies between theaters); 2) estimated probability of a contingency arising in the theater; and 3) vehicle replacement (investment) costs. Specific costs and details of the costing procedure are described in Appendix 3.

Vehicle and Ship Output. The output or contribution which each vehicle can make toward meeting the output mission in which it operates in each theater is tabulated in Appendix 4. Vehicle outputs are computed to show the number of short tons a vehicle type can deliver from ship to shore within the parameters of time and distance specified by the tactical plan of each theater. Ship outputs are the number of vehicles by type which each ship can embark and operate.

The problem has been identified and laid out explicitly in a form subject to analysis. The planner next turns to the task of developing a model.

Construction of the Model

The purpose of the central planner is to recommend a force composition sufficiently effective to meet stipulated output missions (requirements) while minimizing the cost of achieving that pre-determined capability. The method of linear programming is useful in examining problems of optimal allocation of resources to meet given output levels at minimum cost, whether the issue is one of achieving a degree of fire protection, as in the case of a community government, or of

attaining a certain effectiveness for amphibious operations. A linear program cannot, of course, be considered a completely accurate representation of the real world. That is, it is at best an approximation in mathematical terms of the interrelationships of the components of an actual system. The usefulness of linear programming depends (as its name implies) on certain critical assumptions of linearity, proportionality and nonnegativity. To the extent that a system can be presumed to reflect these characteristics, the method is useful, for it permits manipulation of the system to be examined in a way which can add to understanding of the economic and operational interactions of the various components.[1] The problem of determining force levels (selecting a system) of amphibious vehicles and ships needed to meet different theater output missions can be represented by the variables and coefficients of a linear program. A model of the primal and dual of this problem formulated as a linear program is constructed and interpreted by the central planner as follows:

Central Primal.

Let $i = 1,2,\ldots,m$ - an index of operational theaters

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[1] For a discussion of the applicability of linear programming to real world situations see Robert W. Dorfman, Paul A. Samuelson and Robert W. Solow, Linear Programming and Economic Analysis (New York: McGraw-Hill Book Company, Inc., 1958), pp 8-9. For a description of the assumptions made in applying programming to economic and military problems see George B. Dantzig, Linear Programming and Extensions (Princeton, New Jersey: Princeton University Press, 1963), pp 32-35.

Variables.

Q_1 - a vector of amphibious force vehicle levels (Table III, Appendix 2).

Q_2 - a vector of amphibious force ship levels (Table IV, Appendix 2).

$X_1, X_2, \ldots, X_m$ - vectors of theater employment levels of vehicles arrayed by type according to ship class in which embarked and function in theater output mission. One element of this vector identifies the number of vehicles of a particular type operated in a certain output mission from a particular class of ship. (Table V, Appendix 2).

$Q_1, Q_2, X_i \geq 0 \quad i = 1,2,\ldots,m$

Coefficients.

C_1 - a vector of per unit vehicle investment and discounted peacetime operating costs (Table VI, Appendix 3).

C_2 - a vector of per unit ship class investment and discounted peacetime operating costs (Table VI, Appendix 3).

$P_1, P_2, \ldots, P_m$ - vectors of expected theater vehicle attrition costs (Table VI, Appendix 3).

$L_1, L_2, \ldots, L_m$ - vectors of output tonnages defined by theater helicopter, armored amphibian and general amphibious vehicle output missions assigned the amphibious force for delivery of Marine landing force units ashore. One element of this vector is the number of tons which must be transported from ship to shore by the specified means within the time period allowed by the theater tactical plan. (Table II, Appendix 1).

$E_1, E_2, \ldots, E_m$ - matrices of theater per unit vehicle type output tonnage capacities. One element of this matrix is the number of tons a particular vehicle type can transport from ship to shore within the time period allowed by the theater tactical plan. (Appendix 4, Page 87).

K - a matrix of vehicle distribution to different ship classes for purposes of embarkation. One element of this matrix identifies the ship location of a particular vehicle type and the output mission in which the vehicle is operated. (Appendix 2, Page 77).

F - a matrix of vehicle type per unit embarkation requirements by ship class. One element of this matrix is the proportion of the total embarkation capacity of a ship class for a particular vehicle type occupied by one such vehicle. (Appendix 4, Page 90).

I - the identity matrix

Primal Objective Function.

$$\text{Minimize:}\quad C_1' Q_1 + C_2' Q_2 + \sum_{i=1}^{m} P_i' X_i$$

The objective is to minimize the sum of vehicle and ship investment and annual peacetime operating costs and the expected attrition costs of amphibious operations employing these resources.

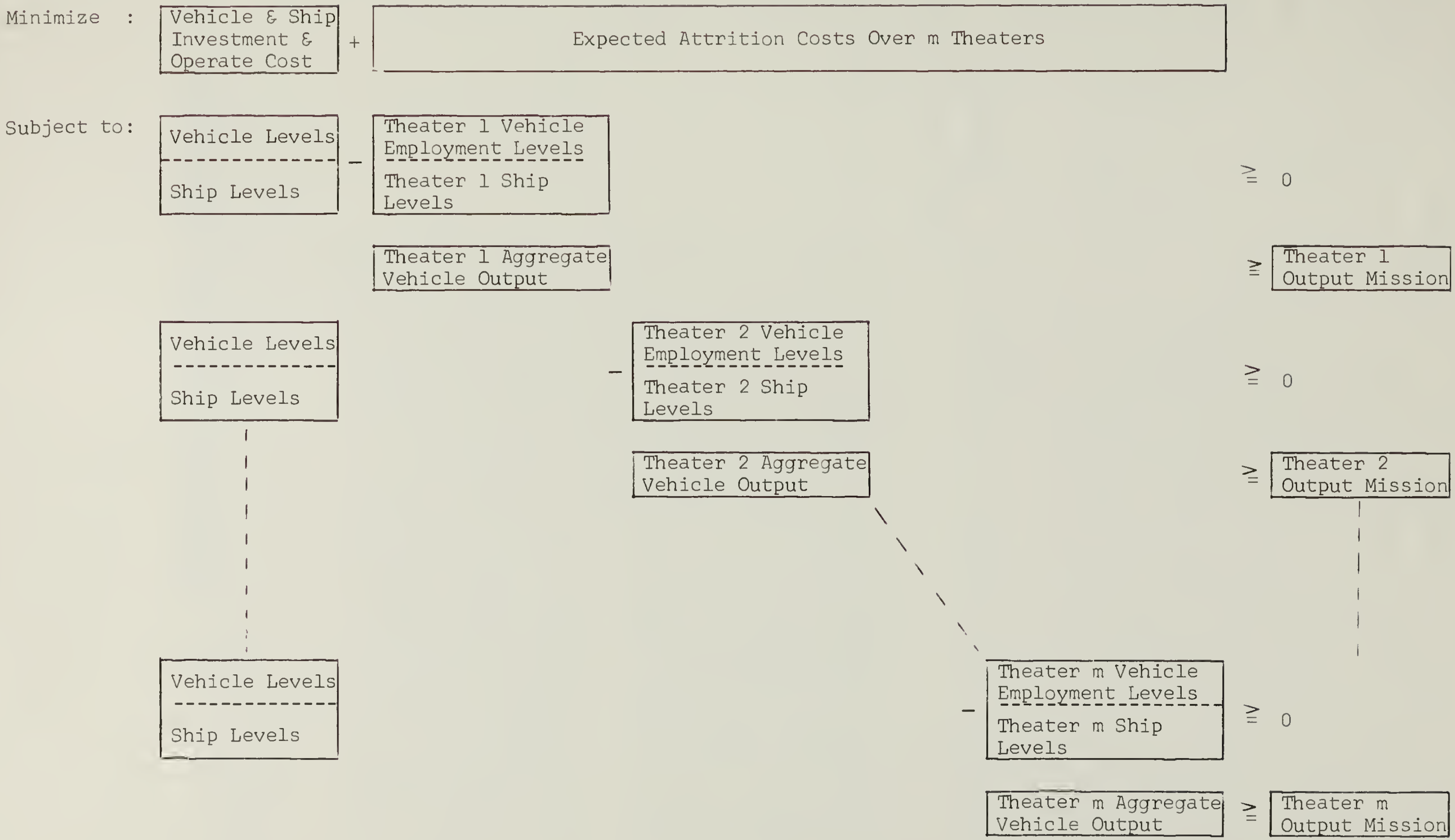
Primal Constraints. The objective of minimizing costs is constrained by functions which insure the availability of sufficient vehicles and ships to meet theater output mission requirements.

(1) Output constraints: $E_i X_i \geq L_i \qquad i=1,2,\ldots,m.$
The aggregate output capacity of the helicopters, armored amphibian and general amphibious vehicles employed in the ith theater is adequate to meet respective output missions.

(2) Vehicle constraints: $IQ_1 - KX_i \geq 0 \qquad i=1,2,\ldots,m.$
The number of vehicles by type in the amphibious force is at least as great as the number employed by type, output mission, and ship class in which embarked in the ith theater.

(3) Ship constraints: $IQ_2 - FX_i \geq 0 \qquad i=1,2,\ldots,m.$
The number of ships by class in the amphibious force is adequate to embark and operate the vehicles employed in the ith theater.

STRUCTURE OF THE CENTRAL PRIMAL


Minimize :
Vehicle & Ship Investment & Operate Cost
+
Expected Attrition Costs Over m Theaters
Subject to:
Vehicle Levels
Ship Levels
−
Theater 1 Vehicle Employment Levels
Theater 1 Ship Levels
≧ 0
Theater 1 Aggregate Vehicle Output
≧
Theater 1 Output Mission
Vehicle Levels
Ship Levels
−
Theater 2 Vehicle Employment Levels
Theater 2 Ship Levels
≧ 0
Theater 2 Aggregate Vehicle Output
≧
Theater 2 Output Mission
Vehicle Levels
Ship Levels
−
Theater m Vehicle Employment Levels
Theater m Ship Levels
≧ 0
Theater m Aggregate Vehicle Output
≧
Theater m Output Mission

Central Dual. Underlying every primal minimization problem is a dual problem of maximization. The variables of the dual linear program completely impute per unit values of different outputs (vehicles and ships in the instant case) to their per unit costs. In the optimal solution, the value of the (maximized) objective function of the dual exactly equals the (minimized) cost of the primal objective function.[1] Dual variables exhibit an economic interpretation which is very useful in the present analysis, as will be shown. The variables, objective function and constraints of the dual are listed and interpreted as follows:

Variables. Dual variables are related to the constraints in order of the primal:

$\pi_1, \pi_2, \ldots, \pi_m$ - vectors of per unit value imputed to each vehicle type in each theater for its contribution in meeting a theater output mission. One element of this vector reflects the reduction in total cost of the amphibious force which would result from the unit addition of a particular vehicle operating in a specified output mission from a particular ship class.

$\varphi_1, \varphi_2, \ldots, \varphi_m$ - vectors of per unit value imputed to each ship type in each theater for its contribution in embarking the vehicles employed in a theater. One element of this vector reflects the reduction in total cost of the amphibious force which would result from the unit addition of a particular ship class.

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[1] For proof of these properties of a linear program see William J. Baumol, Economic Theory and Operations Analysis (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1965), pp 122-125.

$\theta_1, \theta_2, \ldots, \theta_m$ - vectors of value imputed to each ton of the Marine landing force delivered from ship to shore by helicopter, armored amphibian and general amphibious means to meet theater output missions. One element of this vector is the per unit value of an added ton of the Marine landing force delivered from ship to shore.

$\pi_i, \varphi_i, \theta_i \geqq 0 \qquad i=1,2,\ldots,m$

Dual Objective Function.

Maximize: $\sum_{i=1}^{m} L_i' \theta_i$

The objective is to maximize the value of the output mission over all theaters. The objective is equivalent to maximizing the total value of the amphibious force in delivering the Marine landing force ashore in all theaters.

Dual Constraints. The objective of maximizing the value of the force is constrained by functions which require that the sums of the per unit values imputed to the different ships classes and vehicle types over all theaters do not exceed the per unit costs of these resources; and that the value of the service rendered by the force does not exceed the cost of its employment.

(1) Vehicle Value Constraints: $\sum_{i=1}^{m} \pi_i \leqq C_1$

The sum of theater per unit values imputed to any vehicle type does not exceed the per unit vehicle investment and peacetime operating cost.

(2) Ship Value Constraints: $\sum_{i=1}^{m} \varphi_i \leq C_2$

The sum of theater per unit values imputed to any ship class does not exceed the per unit ship investment and peacetime operating cost.

(3) Vehicle Output Value Constraint:

$E_i' \theta_i \leq P_i + K'\pi_i + F'\varphi_i \qquad i=1,2,\ldots,m$

The value of the output capacity (service rendered) of the vehicles in the amphibious force in the ith theater does not exceed the cost of their employment in that theater. One element of the matrix $E_i' \theta_i$ is the product of the vehicle output in tons and the value imputed to one ton of the Marine landing force delivered ashore. The element thus describes the total value of the delivery service rendered (output) by the vehicle in the ith theater. The constraint then relates these values to the "costs" of potential employment of the force where the value expressions $K'\pi_i$ and $F'\varphi_i$ can be considered opportunity costs which accrue from the non-availability of the amphibious force in other theaters while it is employed in the ith theater.

This constraint is re-formulated in the central dual to conform with the dual structure: $-K'\pi_i - F'\varphi_i + E_i'\theta_i \leq P_i$. In this form the constraint states that the net value (opportunity costs subtracted) of the vehicles must not exceed expected attrition costs.

STRUCTURE OF THE CENTRAL DUAL

Maximize :	Value of the Amphibious Force (= Theater Output Missions) Over m Theaters						
Subject to:	Value of Vehicles in Theater 1	+	Value of Vehicles in Theater 2	+,.......,+	Value of Vehicles in Theater m	≤	Vehicle Investment & Operate Costs
	Value of Ships in Theater 1		Value of Ships in Theater 2		Value of Ships in Theater m		Ship Investment & Operate Costs
	Net Value of the Vehicle Output (Service Rendered by the Amphibious Force) in Theater 1					≤	Expected Attrition Costs in Theater 1
			Net Value of the Vehicle Output (Service Rendered by the Amphibious Force) in Theater 2			≤	Expected Attrition Costs in Theater 2
					Net Value of the Vehicle Output (Service Rendered by the Amphibious Force) in Theater m	≤	Expected Attrition Costs in Theater m

Application of the Model

The central planner applies the linear program model in an analytic process which insures that the amphibious force finally selected is the minimum cost force operationally adequate for all theaters. The analytic procedure employed is a decision process which depends upon a discourse wherein resource levels are offered by the planner to theater commanders who subsequently return vectors which measure the marginal value of unit additions to the vehicle and ship elements of the resource vectors. The discourse takes place within a framework of successive solutions to the problem of the central dual and a theater dual linear program (subproblem of the central dual) to be described presently.

The Planners Problem. The problem is to find force level vectors Q_1^o, Q_2^o, and X_1^o, $X_2^o,\ldots,X_m^o$ which minimize total investment, operating and expected attrition costs $C_1' Q_1 + C_2' Q_2 + P_1' X_1 + P_2' X_2 + ,\ldots, + P_m' X_m$ necessary to meet a certain set of output missions L_i, $i=1,2,\ldots,m$. The problem could be solved by the planner, using ordinary linear programming techniques based solely on output missions submitted by the theater commanders and other known information of vehicle output capability, ship embarkation capacity and cost. A more interesting and certainly more realistic method would be to permit the commanders responsible for the conduct of probable amphibious operations to participate in the solution by advising the planner of the operational value each places on the vectors Q_1, Q_2 and L_i $i = 1,2,\ldots,m$

The Theater Commander's Subproblem (Theater Primal). The commander's purpose is to minimize the expected attrition costs of probable amphibious operations $P_i' X_i$ $i=1,2,\ldots,m$, while making

certain that:

(1) $E_i X_i \geq L_i$ - (aggregate output capacity of that portion of the amphibious vehicle force employed in the ith theater is adequate to meet theater output missions).

(2) $K X_i \leq IQ_2$ - (theater allocation of vehicles to the different output missions does not exceed the number available).

(3) $FX_i \leq IQ_2$ - (ships employed in the ith theater to embark and operate the theater amphibious vehicle force do not exceed the number available).

$Q_1, Q_2, X_i \geq 0 \qquad i = 1,2,\ldots,m$

The Market Mechanism. The similarity between the problem of the central planner and the subproblem of the theater commander is evident. Both desire to minimize costs. Both must insure that established theater output missions for the amphibious force are met. Despite their parallel interests and the desirability of a mutual exchange of information, no means by which this dialogue might take place has as yet been identified. One difficulty is that each requires information which can only be provided by the other. Specifically, the vectors Q_1 and Q_2 are initially unknown to the commanders. The planner is also unaware of the relative operational values which the commander might attach to these resource vectors.

A solution to this apparent deadlock is found in the market mechanism which is implicit in the formulation of the central and theater linear programs. The operational values attributed by the theater

commander to the elements of the resource vectors can be viewed as "bids" which the commander will offer for the addition of a particular vehicle or ship to the amphibious force. The planner recognizes that there is no requirement for an actual exchange of dollars. It is only necessary that the decision-making process proceed as if the theater commander bids were in dollar terms. Clearly, if negotiations are conducted in the presence of a market mechanism, the required motivation for minimizing costs and marginal analysis will be active.

To see how the market mechanism operates, consider the logic which might be used by the theater commander to generate the operational value to his theater of additional vehicles or ships. The ith commander is aware that underlying the theater primal is a set of dual variables which will reveal precisely the added potential operational value to the ith theater which will result from the unit addition of a particular vehicle type or ship class to the amphibious force. Suppose that the theater commander defines theater dual variables:

π_i - a vector of bids which measure the operational value to the ith theater of unit additions of vehicle types to the amphibious force $i = 1,2,\ldots,m$.

φ_i - a vector of bids which measure the operational value to the ith theater of unit additions of ship classes to the amphibious force $i = 1,2,\ldots,m$.

θ_i - a vector of values which measure the operational value of a one-ton increase in the size of the Marine landing force delivered from ship to shore in the ith theater.

$$\pi_i,\ \varphi_i,\ \theta_i \geq 0 \qquad i = 1,2,\ldots,m.$$

The theater dual is derived from the theater primal linear program described earlier and takes the form:

$$\text{Maximize:} \quad -IQ_1' \pi_i \ -IQ_2' \varphi_i + L_i' \theta_i \qquad i = 1,2,\ldots,m$$

$$\text{Subject to:} \quad -K' \pi_i \ -F' \varphi_i + E_i' \theta_i \geq P_i \qquad i = 1,2,\ldots,m$$

The solutions of the theater dual for the bid vectors π_i and φ_i are, in effect, dollar values which the theater commander can use to inform the planner of the operational worth he attributes at the margin to the elements of the vectors Q_1 and Q_2. The objective function states that the commander desires to maximize the operational value to the ith theater of the theater output mission (Marine landing force) requirement and hence the net value of the amphibious force in the ith theater. The objective is viewed as the net contribution which the Marine landing force ashore in combat makes to the theater mission. This process is constrained by a function which insures that the operational capability of the force in the ith theater equals at least the cost of its potential employment therein.[1]

Planner-Commander Communication. The analytic means (bids) which permit the theater commanders to participate in the force level decision-making process has been formulated. The validity of this assertion is seen in the properties at the central dual. The central dual is an angular system which will decompose into m theater subprograms and one central master program. The Dantzig-Wolfe algorithm is available for solving linear programs which are decomposable. The decomposition

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[1] See the description of this constraint in the central dual, page 21.

process exhibits properties which can serve the purpose of the central planner. Most important, the procedure can be used as an analytic device for decentralized decision-making.[1]

The theater dual formed by the ith commander is a sub-problem of the central dual. Solution of the theater dual will present vectors of bids attributed by the ith commander to the vehicle and ship elements of any resource vector $\hat{Q}_1$, $\hat{Q}_2$ proferred by the planner.

Inter-Theater Coordination. A means of analytic communication between the planner and each commander using an implied market mechanism has been established. The commanders are, in effect, consumers of the planner's product (resource vectors Q_1 and Q_2). As consumers they will presumably act independently in the "market" supplied by the planner. A means of coordination is required because the planner is interested in maximizing the aggregate value of the amphibious force. The ith theater primal and dual do not accommodate bids of other commanders or the planner's investment and operating cost vectors C_1 and C_2. The ith commander could be expected to maximize the value of the ith output mission independently by attributing bids to the vectors Q_1 and Q_2 which reflect the unique characteristics of the ith theater only. Unfortunately, this procedure may not yield an optimal or even feasible solution. The problem is that theater bids are linked by the central dual constraints $\sum_{i=1}^{m} \pi_i \leq C_1$ and $\sum_{i=1}^{m} \varphi_i \leq C_2$. These constraints are coordinating devices which require that, in the optimal solution, the sum

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[1] For properties of decomposable linear programs see George B. Dantzig, Op. Cit., pp. 448-453.

of the theater bids equal the per unit investment and peacetime operating costs for each vehicle type and ship class. To see why optimality is not possible otherwise, consider some intermediate stage of the process. Since theater commanders are not constrained by the cost vectors C_1 and C_2 some vehicles which are very desirable in all theaters would presumably reflect an aggregate bid greater than the sum of the per unit investment and operating costs. In contrast, vehicle types already provided in sufficient number for all theaters (or which are of no use in a theater) should evoke zero bids. This is because the commanders would attribute no operational value at all to added vehicles of types which are either already adequate to meet an output mission, or are not operationally suitable.

Clearly, more of those vehicles for which the aggregate bid is greater than its cost should be provided, and no vehicle receiving a zero bid should be procured. An optimal solution results only when vehicle and ship operational value bids have been completely imputed in the central dual to the cost vectors C_1 and C_2.

In the optimal solution of the force level problem, a desirable property would obviously be convergence of the values imputed by the planner (solution of the central dual) and the bids attributed by the separate commanders to the resource vectors Q_1 and Q_2. Fortunately, the decomposition algorithm exhibits precisely this property.

If a vehicle force is appropriately selected, it should be possible to induce theater commanders to choose bids which insure that the cost constraints of the central dual are not exceeded.

Suppose that solutions to the value/bid vectors of the central dual are known: π_1^o φ_i^o, θ_i^o, $i = 1, 2, \ldots, m$. The solution of these vectors insures that the primal force level vectors are solved as well Q_1^o, Q_2^o, X_1^o, $X_2^o, \ldots, X_m^o$. The force levels associated with the cost constraints of the dual are the elements of these vectors. Presented with these vectors of force levels, the theater commanders should discover that the value of their amphibious output mission (net value of the amphibious force to the theater) $-IQ_1' \pi_i - IQ_2' \varphi_i + L_i' \theta_i$ is maximized. This is the decomposition principle.[1]

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[1] This exposition is analogous to that used in Edward S. Pearsall, DECOMP - a FORTRAN Coded Subroutine for Solving Decomposable Linear Programs, Institute for Defense Analyses Internal Note N-433(R), Arlington, Virginia: Institute for Defense Analyses).

CHAPTER II

THE DECISION PROCESS

The decentralized decision process formulated by the central planner exploits the decomposable properties of the central dual program. The terminal solution of the decomposition algorithm is, in fact, mathematically optimal. It is the same optimal result which could be obtained by solving the central dual as a straightforward linear program.[1] It is not this aspect of the process which engages the planner's attention, however, (though it gives him confidence in the analytic procedure) as much as it is the intermediate iterative exchange with the commanders which leads to the optimal solution. The interchange takes place in a way which is meaningful in terms of economic theory (most important to the planner) and operational capabilities (first order business of the theater commander).

Description of the Decision Process

It was shown earlier that the planner and the commander each require information which can only be obtained from the other. In particular, values of the resource vectors Q_1 and Q_2 are (initially) unknown to the commander. Similarly, the marginal operational values or "bids" attributed by commanders to these resource vectors are needed by the planner as he seeks a "mix" of vehicles and ships optimal over all theaters. Suppose, in order to set the procedure in motion toward its assured optimal result, the planner transmits provisional resource vectors

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[1] Dantzig, Op. Cit., p. 449.

$\hat{Q}_1$, $\hat{Q}_2$ to the commanders. (The initial provisional vectors might reasonably be solutions to the central primal program for the tactical concept which is the present basis for amphibious force level objectives). In return, the commanders are requested to relay bids which are correlated with the resource vectors, each bid to express the potential operational value measured at the margin, element by element, of an added vehicle or ship to the ith theater.[1]

The planner can reasonably expect something less than complete satisfaction on the part of the commanders with the vehicle and ship levels postulated by the initial provisional vectors $\hat{Q}_1$ and $\hat{Q}_2$. Suppose that this is indeed the case. Bid levels are very high for some vehicles and ships -- commanders are willing to bid handsomely for additional units of these resources. For other ship and vehicle types bids are low or zero -- either nearly enough are available to completely satisfy theater output missions or the particular resource is of no use at all. Further, bid reactions differ between theaters. One vehicle type may receive a zero bid from one commander and a very high bid from another. The vehicle is of no use in the first theater but is very effective in the other. (The LARC-60 can contribute nothing to the output mission of NOREUR, for example, because it cannot meet the minimum ship to shore time/distance requirement, but is of some potential operational value in other theaters; the planner can anticipate a zero bid from NOREUR for this vehicle from the outset).

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[1] As will be shown, the actual manner in which the theater commander plans to employ the amphibious force is of no concern to the planner. The only information required by the planner is the marginal value to the commander (bid) of unit additions of ships and vehicles to the amphibious force.

A last difficulty is that (1) the aggregate bid for additional ships or vehicles of some types exceeds the per unit acquisition and operating cost to the government of the vehicle or ship, and (2) the aggregate bid for the remainder is less than the per unit cost (only under the most fortunate circumstances could the planner expect aggregate bid-cost equality).

The economic significance of these complications is not lost on the planner. Another try is in order. The planner now attempts to improve the operational desirability of his product by offering new provisional resource vectors $\hat{Q}_1$, $\hat{Q}_2$ which reflect higher levels in the amphibious force of the ships and vehicles whose aggregate bids exceed cost and less of those types which fall short. Again the theater commanders respond. Hopefully, the aggregate bid-cost discrepancy will be reduced. And this is how it turns out. As commanders see the increased supply of desirable vehicles and ships their bids for these elements will moderate. These adjustments make intuitive economic sense. The ith commander will bid less at the margin for any vehicle as the aggregate output capacity of that type approaches the ith theater output mission.

The central planner now has two sets of bids, the second set an improvement (from his bid-cost viewpoint) over the first.

The planner at this intermediate stage would presumably continue to repeat the process until aggregate bid-cost equality is achieved. Unfortunately, this happy result may not occur. The implied market mechanism of the linear program may ultimately fail and the aggregate bids either will not converge on the planner's cost vectors or will converge for the wrong reasons. The difficulty lies in the fact that no matter how the planner contorts the provisional resource vectors $\hat{Q}_1$, $\hat{Q}_2$ for some desirable vehicle and ship types, some commanders

may persist in inflated bidding. This is because, in the market mechanism employed by the planner, each commander will act to influence the composition of the force in his own interests.

To see the effect of an inflated bid on succeeding resource vectors offered by the planner, suppose that the ith commander considers the SK-10 highly desirable and offers an (artificially) enormous jth bid for this vehicle. The bid may have the following result:

$\sum_{i=1}^{m} \pi_{SK-10} > C_{SK-10}$ (value of the SK-10 exceeds its cost). When the planner solves the central dual using this bid vector, the SK-10 element of the next vector $\hat{Q}_1$ will reflect increased numbers of the SK-10, a result clearly beneficial to the ith commander but not necessarily to other commanders.

This type of (underhanded) activity by the ith commander is obviously not in the best interests of other theaters. If not constrained, the solution will produce an optimal force for the ith commander which may be of limited value elsewhere. Coercion is implied and the planner resorts to setting targets for the cost minimizing/force value maximizing processes of the theaters. In subsequent iterations, provisional resource vectors will be accompanied by value targets which have the effect of forcing commanders to use resource vectors in a manner which suits the planner's purpose of aggregate optimality for the amphibious force. As will be explained, the analytic procedure of the decision process is formulated in such a way that the value of the commander's bid is subtracted from the value of the amphibious force

(to the theater). If the net value of the amphibious force to the ith theater (bid values having been subtracted) falls short of the value target, the bid is not accepted by the planner. In this fashion, the value target prevents inflated bidding and forces correct use of the planner's resource vectors.

The planner now uses the two bids held and later bids to generate succeeding provisional resource vectors $\hat{Q}_1$, $\hat{Q}_2$ and value targets by forming weighted averages which are designed to improve the (planner's) solution at each iteration.

The coercive process continues over a finite set of iterations until optimal resource vectors Q_1^o, Q_2^o are found. These optimal levels are achieved when every commander exactly meets his target value. It is entirely possible, even probable, that the amphibious force is not optimal for each theater, but it is optimal from the planner's standpoint. It is the minimum cost combination of vehicles and ships which can accomplish the output mission in all theaters.

Analytic Framework of the Decision Process

The interchange of resource level-bid-target information just described takes place within the analytic framework of the linear program model developed in Chapter I with certain modifications which will now be described.

A linear programming problem called the central master program is formed based on the decomposition algorithm which will yield an optimal weighted average of proposed theater amphibious force value vectors $\bar{\theta}$ subject to the constraint that the associated vehicle and

and ship bid vectors do not exceed the central dual cost vectors C_1 and C_2. The central master primal program has three components:[1]

(1) A weighted objective function derived from the objective function of the original central dual and similarly interpreted.

(2) Weighted bid-cost constraints of the central dual.

(3) Constraints on the (unknown) weights, one set for each theater.

The central master primal problem is:[2,3]

$$\text{Maximize: } \sum_{i=1}^{m} \sum_{j=1}^{n_i} L_i' \, \bar{\theta}_{ij} \, \mu_{ij}$$

$$\text{Subject to: } \sum_{i=1}^{m} \sum_{j=1}^{n_i} \bar{\pi}_{ij} \, \mu_{ij} \leq C_1$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n_i} \bar{\varphi}_{ij} \, \mu_{ij} \leq C_2$$

$$\sum_{j=1}^{n_i} \mu_{ij} = 1 \qquad i = 1,2,\ldots,m$$

$$\mu_{ij} \geq 0 \text{ for all } i \text{ and } j$$

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[1] For a thorough description of the decomposition procedure applied to a business firm see William J. Baumol and Tibor Fabian, "Decomposition, Pricing, for Decentralization and External Economics", Journal of Management Science, September 1964, Vol. I, No. 1, p. 1.

[2] This formulation is similar to that used by Edward S. Pearsall, Op. Cit., p. 7.

[3] The identity matrix serves only to dimension the resource and bid vectors in the linear program and has been dropped from the formulation in this chapter for the purpose of reducing notational clutter.

Where n_i - number of proposals of the ith theater

μ_{ij} - the weights associated with the theater bid and value vectors.

$\bar{\pi}_{ij}$, $\bar{\theta}_{ij}$ - jth proposed vehicle and ship bid vectors for the ith theater

$\bar{\theta}_{ij}$ - jth proposed output mission (amphibious force) value vector for the ith theater

i - $1,2,\ldots,m$

j - $1,2,\ldots,n_i$

and $\bar{\pi}_{ij}, \bar{\varphi}_{ij}, \bar{\theta}_{ij} \geq 0$

The weights μ_{ij} are seen to be the only variables. This is because the $\bar{\pi}_{ij}$, $\bar{\varphi}_{ij}$ and $\bar{\theta}_{ij}$ vectors are known values. They are, in fact, current and all previous bids submitted by the theater commanders. At any intermediate stage of the process then, the only true unknowns are the weights. The object of the central master program is the determination of a set of feasible weights μ_{ij},correlated with the latest and earlier bids offered up by the theater commanders. The calculation of the next set of weights will yield a higher valued (lower cost) amphibious force.

The central master program formed by the planner does not include the vehicle output value constraint $E_i' \theta_i \leq P_i + K'\pi_i + F'\varphi_i$ of the original central dual. That is, all information concerning vehicle performance in a particular theater is ignored by the

planner.[1] The planner evidently intends to leave operational matters in the hands of the operational commanders. And this is indeed the case. The planner has taken advantage of the property of the decomposable linear program which permits the commander to employ the amphibious force in any manner most effective in his theater without recourse to the planner concerning these purely operational matters.[2] Such generous behavior by the planner is possible because the same constraint appears in the theater dual derived in Chapter I (p. 26). The way in which this constraint is accommodated, however, is left to the commander as will be explained in the next section.

The Decomposition Procedure in the Decision Process

During intermediate stages of the planner-commander interchange the values of the weights μ_{ij} are disregarded. The purpose of solving the central master program is instead the generation of new provisional resource vectors $\hat{Q}_1$ and $\hat{Q}_2$.

Solution of the dual of the central master yields the resource vector information needed for the next iteration of bids. The central master dual is formed:

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[1] This statement is not completely accurate in that planner-commander coordination external to the analytic process is necessary in the determination of expected attrition costs. Probability and attrition rate inputs to these costs must be agreed upon and fixed. Otherwise, commanders (very close to the threat) could escalate these values unreasonably and thereby inflate requirements (see Appendix 3 for derivation of expected attrition costs).

[2] See Baumol and Fabian, Op. Cit., p. 8.

Minimize: $C_1'\hat{Q}_1 + C'\hat{Q}_2 + \sum_{i=1}^{m} \psi_i$

Subject to: $\bar{\pi}_{ij}'\hat{Q}_1 + \bar{\varphi}_{ij}'\hat{Q}_2 + \psi_i \geq (L'\bar{\theta}_{ij})'$ for all i and j

where $\hat{Q}_1$ - a vector of provisional vehicle levels

$\hat{Q}_2$ - a vector of provisional ship levels

$\psi_1, \psi_2, \ldots, \psi_m$ - a planning target which measures the marginal increase in aggregate value of the amphibious force over all theaters resulting from a unit increase in net value of the force to the ith theater.

The role of the planning target ψ_1 is a coercive one and its interpretation will perhaps be better understood after a review of the bid-generating activity of the theater commander. Recall that the ith commander formulated his theater primal subproblem:[1]

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[1] It was pointed out earlier that probability and attrition values used in computing expected attrition costs P_i must be coordinated outside the analytic process. Theater commanders, however, are not likely to view the probability of amphibious operations with quite the same detachment as the planner. An alternative procedure could be adopted which would partially resolve this difficulty in a way which may be useful to the planner in deciding on relative priorities between theaters. The coordination problem can, in fact, be finessed completely by permitting commanders to participate in the bidding procedure as if the prospect of an operation in each theater were certain, i.e. compute expected attrition costs using unit values of probability. The planner can retain judgment at his (national) level as to the relative probability of operations (and hence priorities) among the various theaters. To see why this is so, consider the ith theater subproblem after the jth bid:

$$-K'\bar{\pi}_{ij} - F'\bar{\varphi}_{ij} + L_i'\bar{\theta}_{ij} = P_i'X_i$$

The duality theorem states that the (maximized) net value of the amphibious force in the ith theater exactly equals its (minimized) expected attrition cost. But expected attrition cost P_i is a linear function of the probability of an amphibious operation in the ith theater (see Appendix 3). Since K, F and L_i are constants and P_iX_i is also a linear function of probability, the planner can legitimately scale the theater bids by any values he may choose, to reflect the relative priority to be given the requirements of the different theaters.

Minimize: $P_i X_i$ $\quad i = 1,2,\ldots,m$

Theater Dual Variable

Subject to: $K X_i \leq Q_1$ $\quad \pi_i$

$F X_i \leq Q_2$ $\quad \varphi_i$

$E_i X_i \geq L_i$ $\quad \theta_i$

Solution of this linear program using any provisional resource vectors $\hat{Q}_1$, $\hat{Q}_2$ will produce the theater bid vectors $\bar{\pi}_{ij}$, $\bar{\varphi}_{ij}$, $\bar{\theta}_{ij}$ for the next bid submission to the central planner (as explained in Chapter I, page 26).

The planning target ψ_i is associated with the weight constraint $\sum_{j=1}^{n_i} \mu_{ij} = 1$ of the central master program. This constraint serves to require that the variables of the primal program are true averages of the ith theater bids (Σ $\bar{\pi}_j \mu_j$, $j = 1,\ldots,n_i$, for example). ψ_i must be interpreted as the marginal value to the planner of a 100% increase (doubling the weighted average of the theater bid) in the net value of the amphibious force operating in the ith theater. Thus it is the marginal increase in aggregate value of the amphibious force to the planner which results from a shift of the output potential of the force in the interests of the ith theater (include more of the ships and vehicles for which the ith commander is bidding). The target ψ is used by the planner to insure that commanders "play the game", i.e. generate legitimate bids using the provisional vectors $\hat{Q}_1$, $\hat{Q}_2$.

To see how the target operates consider again the dual of the ith theater primal:

$$\text{Maximize:} \quad -Q_1' \pi_j - Q_2' \varphi_j + L'\theta_j \qquad j = 1,2,\ldots,n_i$$

$$\text{Subject to:}^{1} \quad E'\theta_j \leq P + K'\pi_j + F'\varphi_j \qquad j = 1,2,\ldots,n_i$$

The theater commander solves the dual to obtain the next bid but before he sends the vectors forward he must peform a test using the value target ψ_i. Now the target $\hat{\psi}$ is significant to the commander because it is, in effect, the net value of his objective function (net value of the amphibious force in his theater) for the previous iteration of bids. This is so because ψ_i is the value to the planner of a 100% increase in net value in the ith theater. Hence $\hat{\psi} = -\hat{Q}_1' \bar{\pi}_j - \hat{Q}_2' \bar{\varphi}_j + L_1' \bar{\theta}_j$. The test is $-\hat{Q}_1' \bar{\pi}_{j+1} - \hat{Q}_2' \bar{\varphi}_{j+1} + L' \bar{\theta}_{j+1} \geq \hat{\psi}$. That is, in the succeeding round of bidding, the commander must at least achieve the previous net value of his maximized dual objective function. This comparison forces the commander to choose bid values (avoid inflated bids) such that the total value of the amphibious force in his theater exceeds the total amount he is willing to bid for his potential use of the amphibious force by at least the target value (which he attained in the previous iteration). If he attempts to cheat by inflating bids artificially to influence the force composition in his favor, the (negative) effect of such a bid will cause him to fall short of the target and his bid will not be accepted by the planner.[2] After the comparison is successfully made, the commander forwards the new bid vectors to the planner.

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[1] The theater dual includes the "missing" constraint of the original central dual, which was excluded from the central master program.

[2] Similarly, if the commander submits artificially reduced bids, he foregoes his opportunity to influence the composition of the force (relative to other commanders). There is a presumption of honesty in the entire bidding process.

The Optimal Solution

The planner-commander dialogue is not an endless interchange. After a finite number of iterations, the process terminates in an optimal solution. The target vector ψ is used by the planner to identify the point at which this solution occurs. At some stage of the process, the planner will have a set of proposed bid and value vectors which do not violate theater cost constraints and which exceed or at least meet the previous target ψ_i. (If this is not the case in any theater, the planner recommends retirement for the commander). Suppose that after j bid proposals, m - 1 commanders have just met their targets

$$\sum_{i=1}^{m-1} -\hat{Q}_1' \bar{\pi}_{ij} - \hat{Q}_2' \bar{\varphi}_{ij} + L_i' \bar{\theta}_{ij} = \hat{\psi}_i$$

and that the ith commander has exceeded his target

$$-\hat{Q}_1' \bar{\pi}_j - \hat{Q}_2' \bar{\varphi}_j + L' \bar{\theta}_j > \hat{\psi}_i$$

The amount by which the ith commander has exceeded his target has been shown to be a measure of the marginal increase in the aggregate value of the amphibious force which the ith commander's jth bid proposal offers the planner. The planner will gain by revising the composition of the force according to the pattern recommended by the ith commander by incorporating his jth proposal and recomputing the central master program.[1]

Iterations are continued until all targets are exactly met. At this point optimality criteria are satisfied:

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[1] See Baumol and Fabian, Op. Cit., p. 14.

(1) All theaters have just met their value targets -

$$-\hat{Q}_1' \bar{\pi}_{ij} - \hat{Q}_2' \bar{\varphi}_{ij} + L_i' \bar{\theta}_{ij} = \hat{\psi}_i \qquad \text{for all } i \text{ and } j; \text{ and}$$

(2) Aggregate bid-cost equality is achieved -

$$\sum_{i=1}^{m} \bar{\pi}_i = C_1 \text{ and } \sum_{i=1}^{m} \bar{\varphi}_i = C_2$$

The central master is then solved for the optimal weights μ_{ij}^o $j = 1,2,\ldots,n_i$ which simultaneously yield optimal resource vectors Q_1^o and Q_2^o (via the dual of the central master).[1]

No adjustment in composition will increase the aggregate value of the amphibious force (reduce its cost) and the value of the force is maximized (expected attrition cost minimized). The force composition Q_1^o, Q_2^o to be recommended for acquisition has been identified and the decision process is complete.

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[1] Solution of the theater primal programs, using the resource vectors Q_1^o, Q_2^o will yield optimal theater vehicle employment vectors X_i^o, $i=1,2,\ldots,m$ as well. It is emphasized, however, that this is a mathematical result only. The vehicle and ship levels reflected in the optimal solution are imposed on the commanders. They must plan for future operations based on the "mix" selected by the planner (until the next review of the amphibious force structure).

CHAPTER III

SOLUTION

The central planner is now prepared to summarize the results of the cooperative decision process in the form of amphibious force level objectives to be recommended. The decomposable linear program has been solved using known investment and operating cost information and successive bids generated by theater commanders. The commanders in the process have taken account of the operational effectiveness of the amphibious force to be recommended and found it to be adequate.[1]

Results

Tabulated below are the amphibious force objectives yielded as optimal solutions for the vectors Q_1 and Q_2.

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[1] The decomposable linear program model was programmed and run on a CDC 1604 computer using the DECOMP subroutine previously cited and the data in Appendixes 1-4.

TABLE X

RECOMMENDED AMPHIBIOUS FORCE

Vehicle/ Ship[2]	Output Mission	Central Master[1] Solution (Q_1^o, Q_2^o)	Theater Subproblem Solution (Employments Levels)			
			SEASIA	MIDEAST	NOREUR	CI[3]
CH53	Hcptr	340	340	132	241	166
CH46	"	0	0	0	0	0
LVTP	Armored Amphib.	0	0	0	0	N/A
LVTX	"	600	300	600	600	N/A
SK-10	Gen. Amphib.	124	27	76	124	32
LCA	" "	0	0	0	0	0
LARC-60	" "	0	0	0	0	0
LCU	" "	0	0	0	0	0
LCM-8	" "	0	0	0	0	0
LPH (7)		14	14	6	10	7
LPD (16)		16	14	5	16	12
LSD (5)		24	2	24	24	2
LST (18)		18	0	0	18	0

Inspection of this table provides an insight into the cost minimizing/value maximizing process of the linear program. The results are intuitively satisfying from both the planner's (cost) and the commander's (operational) viewpoint. For example:

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[1] Force to be recommended.

[2] Figures in parentheses adjacent to ship classes are the levels available before 1970 treated as sunk costs in the analysis.

[3] Armored amphibians are not required in the ship-to-shore assault role in CI.

1. All theater output missions can be accomplished by the force selected.

2. No overall dominance is exhibited in the theater employment levels. NOREUR dominates MIDEAST and CI, but not SEASIA. MIDEAST and SEASIA dominate only CI.

3. The LSD is the most attractive ship because it is an efficient carrier of the SK-10, the vehicle selected to perform the entire "general amphibious" output mission. Since add-on LSD's were needed for NOREUR, it is cheaper to use some of these added ships in MIDEAST to embark SK-10's and LVTX's than some of the LPD's and _all_ of the LST's already in the force by 1970.

4. No additional LPD's or LST's are included in the solution and LST's are used only after all LPD's are employed in NOREUR. The linear program has used all available ship assets before "buying" additional ships (LSD's and LPH's).

5. Although the SK-10 is relatively expensive (exceeded only by the LCU which has been penalized for its non-amphibious characteristics), its superior performance dominates the general amphibious output mission.

Vehicle/Ship Employment

The way in which the vehicle force is embarked in the ships in different theaters is provided by the solutions of the theater subproblems for the employment vectors X_i $i = 1,\ldots,4$.

TABLE XI

VEHICLE/SHIP EMPLOYMENT[1]

Ship	Comp'tble Vehicle	Output Mission	SEASIA		MIDEAST		NOREUR		CI	
			Veh	Ship	Veh	Ship	Veh	Ship	Veh	Ship
LPH	CH53	Hcptr	323	14	126	6	229	10	156	7
	CH53	(0)	17		6		12		10	
LPD	LVTX	Armored Amphib.	300		0		6		0	
	SK-10	Gen. Amphib.	20	14	10	5	31	16	24	12
	SK-10	(0)	0		0		0		0	
LSD	LVTX	Armored Amphib.	0		600		0		0	
	SK-10	Gen. Amphib.	0	2	43	24	55	24	0	2
	SK-10	(0)	7		23		38		8	
LST	LVTX	Armored Amphib.	0	0	0	0	594	18	0	0

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[1] (0) indicates that the vehicle performs also in the outsize output mission sub-category (See Table II, Appendix 1).

The Optimality Check[1]

To see how all this comes about and to show that the decentralized decision process has, in fact, produced an optimal solution, recall the optimality criteria that: (1) theater commanders just meet operational value planning targets and (2) the aggregate bid (operational value) equals per unit investment and operating costs.[2]

TABLE XII

FORCE VALUE-PLANNING TARGET EQUALITY

Theater	Theater Net Force Value	Central Master Planning Target
SEASIA	37.1913	37.191
MIDEAST	17.2466	17.247
NOREUR	25.1052	25.105
CI	13.6801	13.680

Table XII shows that commanders have exactly met the planning target generated by the solution of the dual of the central master program after the last round of bidding. This result assures the planner that no change in the composition of the force will improve its aggregate value (reduce its cost), and hence no repetition of the bidding process is required and an optimal solution has been determined.

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[1] It is emphasized that the entire linear program problem was solved in an iterative process by one computer. The computer used 14 successive solutions of the central master program and the theater subproblems to yield the optimal result. This is a property of the decomposition algorithm. The theater subproblem, however, can be extracted from the computer program, and the central master will accept bid information computed externally, subsequently providing the required resource vectors and targets for the next iteration. That is, the theater subproblem can be solved in the theater.

[2] Chapter II, page 42.

TABLE XIII

VEHICLE/SHIP AGGREGATE BID-COST EQUALITY[1]

	BIDS							
Vehicle/Ship	SEASIA	MIDEAST	NOREUR	CI	$\sum_{i=1}^{4} \pi_i^o, \varphi_i^o$	$\leq$	C_1, C_2	Q_1^o, Q_2^o [2]
CH53	4.2	0	0	0	4.2	=	4.2	340
CH46	2.80	0	0	0	2.80	<	2.835	0
LVTP	.0388	.3357	.0017	0	.3762	<	.378	0
LVTX	0	.2446	.0288	0	.2734	$\cong$	.273	600
SK-10	0	0	6.030	0	6.03	=	6.03	124
LCA	0	0	.650	0	.650	<	.652	0
LARC-60	.725	0	0	.0047	.7297	<	.733	0
LCU	1.467	0	5.790	.0266	7.2836	<	7.285	0
LCM-8	0	.0413	1.857	.0063	1.9046	<	1.905	0
LPH	79.31	0	0	0	79.31	=	79.31	7
LPD	0	.0020	34.15	0	34.1520	<	79.82	0
LSD	0	.0068	66.45	.0073	66.464	$\cong$	66.45	19
LST	0	.0068	16.62	0	16.6268	<	46.77	0

These results reflect additional properties desired by the planner in the optimal solution. The sum of the bids for six vehicle types and two ship classes are less than related per unit investment and operating

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[1] The solution, which does not "buy" LPD's or LST's, is partially misleading from an operational standpoint. This is because these ships function in other missions not considered by the analysis. The LST, for example, is required for its beaching capabilities to land large numbers of tracked and wheeled vehicles of the landing force quickly. The LPD is a major troop transport and also operates helicopters. An alternative procedure could be to include the LST as a competitor in the "general amphibious" output mission (although these ships are not normally employed in the very early stages of the assault).

[2] Less ships whose costs are treated as sunk.

costs and no additions of these vehicles and ships were recommended in the optimal solution. Similarly, all vehicles and ships reflecting aggregate bids equal to cost were recommended in sufficient quantity. Two interesting points can be made from the table:

1. SEASIA (the largest helicopter user) is bidding at the margin the entire cost of the LPH and the CH53; all other theaters having long been satisfied as is seen in Table X.

2. NOREUR (the heaviest user of general amphibious vehicles) is bidding the entire cost of the SK-10 and LSD while still offering (low) bids for LPD's and LST's and some general amphibious vehicles. Clearly, the LSD/SK-10 combination is preferred.

Solution by Individual Theater

In order to investigate the effect of optimizing the amphibious force for one theater (plan) only, the model was solved for each theater individually (3 theaters deleted). Table XIV below compares the results of individual theater optimization with the combined optimal solution of Table X.

TABLE XIV

COMPARISON OF INDIVIDUAL AND COMBINED SOLUTIONS

(Individual/Combined)

Vehicle/ Ship	SEASIA	MIDEAST	NOREUR	CI
CH53	340/340	132/132	241/241	166/166
CH46	0/0	0/0	0/0	0/0
LVTP	139/0	0/0	0/0	N/A
LVTX	105/300	600/600	600/600	N/A
SK-10	0/27	76/76	124/124	0/32
LCA	0/0	0/0	0/0	0/0
LARC-60	98/0	0/0	0/0	89/0
LCU	0/0	0/0	0/0	0/0
LCM-8	0/0	0/0	0/0	0/0
LPH	14/14	6/6	10/10	7/7
LPD	16/14	16/5	16/16	16/12
LSD	5/2	2/24	24/24	3/2
LST	18/0	18/0	18/18	0/0

Examination of the table shows that, except for NOREUR, the forces are different between the individual and combined cases. The table provides some useful information:

1. In the individual case the theaters with the lesser general amphibious output missions used vehicles of lesser performance capability. SEASIA and CI used the LARC-60, the least productive but also the vehicle with the highest output/cost ratio. (This result is easily justified when it is recalled

that sufficient time (4 hours) is available in the SEASIA and CI tactical plans for this vehicle to accomplish the general amphibious mission in these theaters). In the combined case, the need for SK-10's in MIDEAST and NOREUR forced use of this vehicle in SEASIA and CI in lieu of the LARC-60.

2. The inter-theater effects of the combined solution are quite apparent with respect to ships also. When theater plans are solved individually, available LPD's and LST's are used completely and no add-on LSD's are employed in SEASIA, MIDEAST or CI. (The NOREUR requirement exceeds this capacity and "buys" LSD's as expected). In the combined solution, however, it is less costly to use add-on LSD's in MIDEAST to substitute for some LPD's and all LST's.

Sensitivity Check

The most sensitive aspect of the costing procedure is the influence of the values asserted as the probability of the various theater contingencies arising. Expected attrition costs P_i $i = 1,\ldots,4$ are linear functions of probability (see Appendix 3). In order to test the effects of changes in these values (and, incidentally, to test for possible complete dominance of the solution by the NOREUR tactical plan), the problem was solved in 5 increments which bracketed the probability assigned NOREUR in the basic combined solution (.05) as follows:

$$.02 \rightarrow .04 \rightarrow .06 \rightarrow .08 \rightarrow .10$$

These changes in probability of operation in NOREUR produced no change in the overall force structure. The effect was simply to raise total costs by an increment of $16.7 million dollars. Acquisition costs of investment and operation clearly dominate the solution in this probability range.

Force Level Implications of Results

The foregoing results suggest that optimizing the amphibious force for one tactical plan (one set of parameters) will not yield a force which is optimal over a range of possible contingencies. Table XIV shows, in fact, that radically different forces are required to meet theater contingencies taken individually (one amphibious force per theater) or collectively (one amphibious force for all theaters).

CHAPTER IV

CONCLUSIONS

Contrast of Model Interpretations and Interactions

The present application of the decomposition process to decentralized decision-making depends upon an interpretation very different from that commonly given in the literature to decomposable linear program structures. The interaction between the problem of the planner and the subproblem of the commander takes place in a way precisely opposite that of the usual firm-subdivision dialogue. The usefulness of the decomposition process is customarily demonstrated in a setting in which the firm desires to allocate firm-wide resources to subdivisions in a manner which will insure that subdivision activities are operated at levels which achieve some pre-determined goal *of the firm* at minimum cost.[1] In contrast, the decomposable model suggested in this paper is directed toward attainment of the goals of *subordinate elements* at minimum cost. The motivation attributed to the central authority in this context is one of developing an optimal resource structure which will, when required, provide an operational capability to any subordinate element adequate to accomplish the element mission.

The iterative process emphasizes value at the point of use. It does this by permitting the resource consumer (commander) to influence the product of the resource supplier (planner). The

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[1] See Baumol and Fabian, *Op. Cit.*, and Dantzig, *Op. Cit.*

planner-commander relationship is seen to be the reverse of the ordinary practice in which the firm (ultimately) dictates the product of the firm's subdivisions. In the present case, resource levels are proferred to the commanders who then determine the relative effectiveness of the different resource elements (to the commander) and respond with bids which measure marginal operational value. The aggregate commander bid reaction thus (in the optimal solution) dictates product levels to be adopted by the planner via the implied market mechanism of the linear program. The influence of the planner is used in coordinating the bidding process to insure that the aggregate bid yields a minimum cost force which is optimal from a comprehensive viewpoint, i.e. it is the minimum cost force which can perform the mission in all theaters. The result is entirely plausible from an economic viewpoint. At the optimal solution, bids for the amphibious force accepted by the planner (aggregate operational value of the force) exactly equal its per unit cost (an outcome analogous to the interaction of supply and demand forces in a market impelled toward equilibrium).

Usefulness of Present Procedures

The present procedure for determining amphibious force levels assumes that one general tactical plan is adequate for the development of future force objectives. The solution suggests that this procedure is questionable at best. Each of the four theaters included in the analysis required a different mix (individually or collectively). Further, no force developed for one theater could accomplish the

output mission of each of the others within the parameters established by the theater commanders. The one-plan approach to determination of ship and vehicle requirements does not appear to be a sound basis for future programming of amphibious force levels.

General Applicability of the Decentralized Decision Process

The analytic elements of a force level problem set out in this paper are the same inputs which are used in many of the programming procedures of the Department of Defense. Decisions are, in fact, made within the Department based on an information flow which is similar to the planner-commander dialogue postulated. The Secretary of Defense and the Service and joint components of the Department generate and evaluate force level proposals at least annually. The process requires an enormous negotiating effort -- some of it analytical, but much of it verbal and subject to interpretation.

The decision process suggested can substitute an analytic technique, rigorously supported by mathematical and economic theory, for at least some portion of the procedure which relies on semantics and subjective judgment (experience). The planner-commander dialogue of the decomposition process provides for explicit expression of operational value and cost criteria in terms which are not open to interpretation in the sense that there is no doubt about what is meant (by a zero bid, for example) or how much.

Improvement and Expansion of the Model

The simple model developed clearly could not function as an instrument for decision-making in its present form. It should (and can) be improved by adjustment to accommodate other characteristics

of amphibious resources and operations which bear on the overall effectiveness of the amphibious force. The most critical deficiency in the model is that it does not account for the embarkation requirements of the Marine landing force in an explicit fashion. Ships are generated only to accommodate the ship-to-shore vehicle force. It turns out that these ships are probably adequate to embark the initial assault elements of the landing force with substantial residual capacity remaining, but this result is by default. The model should be improved to accommodate vehicle force/landing force embarkation inter-action precisely.

The model can easily be extended to include such additional performance considerations as vehicle cross-country mobility, endurance, service life, and so forth.

Additional Uses of the Model

It its present form, the decomposable model exhibits other properties useful to the central defense planner which permit analysis of the effect of change in elements other than the variables of the program. The coefficients are subject to parametric variation, for example. This feature of the model permits the planner (or commander) to explore the impact on the force structure of changes in attrition, investment and operating costs, vehicle output capacities and so on. Another revealing aspect of a parametric analysis would be the examination of the effect of changes in output missions (requirements). It is these output requirements which dictate total costs. If requirements were not approved (as assumed) the planner could present

efficient alternative force structures and costs derived from such an examination (leaving to the policy-maker the decision as to the optimal force structure to be adopted).

The model can be adjusted to reflect shifts in the strategic environment and related priorities. This is accomplished by changing the relative probabilities of conducting operations among theater (scaling commander's bids). As the probability of an operation in any theater increases (other remaining fixed) so does the relative influence of the theater on the composition of the force, and hence its priority is in effect increased.

Summary

The decomposable model constructed in this paper is believed applicable in general to solution of force level problems which include basic ingredients of (1) operational capability to meet (2) a range of requirements at (3) minimum cost. It is a device for cooperative decision-making which can assist in insuring that the (minimum cost) force structure decided upon is completely feasible from an operational point of view because it places in the hands of the responsible user an analytic means for making his views known with certainty to the decision-maker.

APPENDIX 1

THEATER CHARACTERISTICS AND TACTICAL PLANS

In order to determine the composition of a force of amphibious vehicles and ships suitable for use over probable contingencies, the military planner must consider the effects on amphibious warfare of: (1) differences in the operational environment between geographical regions, and (2) probable opposition in each region. Most of the probable requirements for large-scale amphibious operations can be placed in one of the three types of contingencies according to the dominant terrain of the area: jungle, desert, and cultivated/industrial.

For purposes of visualizing and subsequently evaluating the relative influence of different regional environments on the desired amphibious force, four operational theater contingencies which require U.S. preparedness for amphibious assault operations are postulated:

(1) Southeast Asian theater (jungle)
(2) Middle Eastern theater (desert)
(3) Northern European theater (cultivated/industrial)
(4) Counterinsurgency[1]

Each of the theaters exhibits certain operational characteristics which influence both the type of Marine force to be landed, and the way in which it is desired to land that force. Primary factors which affect tactical planning for each area are:

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[1] The fourth contingency for operations in an insurgency environment is geographically unrestricted.

(1) The sea approaches and beach configuration in the objective area.

(2) The terrain in which the Marine landing force will operate.

(3) The amount and quality of probable enemy opposition.

In the discussion which follows, it is assumed that the amphibious assault is delivered from a Naval amphibious task force composed of Naval amphibious ships and vehicles and a Marine landing force of combined arms, air and ground. It is further assumed that the landings are opposed and occur in circumstances of active combat.

Southeast Asian Theater (SEASIA)

Consider first the amphibious environment of the SEASIA theater. Open sea approaches to probable objective areas in this region of the world are relatively unrestricted and ample sea maneuver room is available. In addition, opposition to the seaward activities of the amphibious assault is not expected, since no significant opposing Naval force exists in this area. Many beaches suitable for landing amphibious vehicles are available, although major difficulty is presented by the limited water depth of the close-in approaches to the land. Gradually shoaling waters surrounding the beaches have the effect of forcing the amphibious ships to launch assault vehicles from distances well out to sea, often 10 miles or more. The terrain in which the landing force must operate is largely thick jungle, with very few roads, and is impassable to heavy military vehicles. Clearings resulting from crop cultivation are frequent in the jungle canopy, however. The characteristic lack of ground mobility in this theater

and the availability of clearings dictate primary reliance on the helicopter for tactical maneuver. With respect to the third factor listed above, armed forces in this region of the world consist mainly of large infantry formations which characteristically conduct a mobile defense taking advantage of the jungle canopy, but offering little initial opposition in the beach area. These types of forces can successfully be taken under attack by helicopterborne Marine forces supported by tactical aviation (a capability not found in armed forces of SEASIA).

Middle Eastern Theater (MIDEAST)

Conditions for amphibious assault in the MIDEAST theater are in direct contrast to those of SEASIA. Here open sea approaches are more restricted, and available sea maneuvering space reduced in the Mediterranean, Red, and Arabian Seas. Although Naval forces in this region are limited and pose no important threat to the task force at sea, the presence of substantial tactical aviation opposition coupled with more restricted sea space requires the amphibious task force to launch the assault from greater distances, enabling the task force to maneuver evasively, if required. Numerous beaches are available, and the close-in approaches more suitable for deep-draft amphibious ships than those of SEASIA. Ground mobility for the heavy tracked vehicles of the Marine landing force is unlimited. The type of opposition to be expected in MIDEAST influences the composition of the landing force to a greater extent than in SEASIA. In this instance, light helicopterborne forces are less effective against the armored formations of nations in this area than against the infantry forces typical of SEASIA.

Further, the significant tactical aviation capabilities of some Middle Eastern nations reduces the utility of the helicopter. There is also some possibility of a requirement for direct assault over beaches defended by mines, obstacles and entrenched forces.

The amphibious environment in the MIDEAST theater thus suggests emphasis on a capability to land Marine forces which include substantial armored amphibian, tank and artillery capability with a derivitive requirement for an amphibious force able to land the heavy units characteristically employed in desert warfare.

Northern European Theater (NOREUR)

Amphibious operations in the NOREUR theater are the most difficult of all four postulated contingencies. Here available beaches are severely limited. Maneuver space in the North Sea is greatly reduced in comparison to the other theaters. Substantial Naval as well as tactical aviation opposition can be expected at sea.[1] These conditions require that the assault be launched at maximum possible distance in a minimum time period to permit the Naval task force to maneuver to protect itself while continuing to support the Marine landing force ashore. Heavily armored forces can be anticipated in probable objective areas, supported by powerful tactical aviation elements. A further complication, not exhibited in the MIDEAST or SEASIA theaters is the probability of entrenched beach defenses protected by extensive water and land mine fields and other obstacles. Armored amphibians must be provided in adequate numbers for direct assault of these defended beaches. The Marine landing force must be heavily weighted by added tank and artillery units for landings in this region. Because of the probability of strong beach defenses, a capability

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[1] The maneuver space requirement is closely linked to the submarine threat.

to avoid shore areas must also be retained. In amphibious assault operations the employment of helicopterborne forces which can overfly or flank defended beaches provides the tactical flexibility needed to reduce the effectiveness of enemy opposition in the beach area. As a consequence of the quality of the expected enemy in Northern Europe, in combination with a propensity to defend at the beach, the conduct of amphibious assaults in Northern Europe requires the availability of both helicopter and heavy surface assault capabilities in the amphibious force.

Counterinsurgency (CI)

The preceding theater contingencies are related to geographical areas and are presumed to require preparedness for landing under circumstances of conventional combat. There is one type of requirement for amphibious operations not provided for in these contingencies, however, which has a very high probability of occurrence. The need for an amphibious force capable of dealing effectively with problems of insurgency during the period 1970-1979 can be anticipated. Because of the many areas in which insurgency might arise, it does not seem reasonable to relate such a requirement to a single geographical area (theater). The multitude of possible types of insurgency would also appear to be prohibitive for analysis. Rather, a single, generalized counterinsurgency theater is postulated for the purpose of identifying requirements for a wide range of operations in such an environment. Military counterinsurgency operations encompass a very broad spectrum of military activity ranging from quelling civil disturbance to defeating highly organized compaigns of terror, sabotage and subversion. Situations of this kind do not require the firepower of the ordinary

Marine landing force which is capable of breaching established defenses or destroying heavy armored units. Indeed, the need is for a far less obtrusive force which can do the job with dispatch and be withdrawn as quickly as it is landed when local forces are able to resume control.

The composition of a counterinsurgency force emphasizes mobility and capabilities for quick reaction, occupation of many locations simultaneously, rapid communications, and effective intelligence. The helicopter is the most useful means for providing light, fast-moving units which exhibit these characteristics. Concerning the ship-to-shore movement, the most important factors are the ability of the force to seize key locations over a large area, and to establish an effective base of operations for command and control and logistic support ashore, all in a short period of time. Armored amphibian vehicles, though useful in counterinsurgency operations, are not required in the sense of assault under conditions of active combat. In this context, the amphibious operation is not an assault, but a rapid deployment ashore of forces preponderantly infantry in nature. Probable insurgency develops a requirement for a substantial helicopterborne capability with a greatly reduced need for an amphibious force capable of rapidly delivering all of the heavy firepower and logistic support elements of the normal Marine landing force. The surface portion of the ship-to-shore movement in the counterinsurgency environment is devoted primarily to quick and efficient off-loading of logistic and headquarters units and supplies with only small security forces normally required in advance. Further, ship-to-shore distances are typically reduced, and requirements for security at sea correspondingly small.

Representative Theater Tactical Plans

The foregoing comparison of the factors influencing the conduct of amphibious operations in the four scenarios furnishes the necessary background for development of representative tactical plans which permit translation of the theater requirement for an assault capability into terms of specific forces which can be measured and evaluated for purposes of operational and economic analysis. These tactical plans are the output mission requirements to be used by the central planner in the force level decision-making process.

Tactical planning to accomplish the mission of an amphibious operation includes: (1) the selection of physical military objectives to be seized or destroyed, such as terrain features and enemy forces and installations which are identifiable on the ground; (2) the determination of the composition of the Marine landing force necessary to achieve these objectives; and (3) the means and timing of delivering the landing force to accomplish the mission. Although many other factors must be considered in amphibious planning, the purpose of the present analysis is served by developing those elements of the tactical plan related directly to the ship-to-shore movement which require answers to the following questions:

(1) How shall the landing force be structured (organization, number of troops, types of equipment, supplies)?

(2) How shall the elements of the landing force be landed?

(3) How much time is available for the ship-to-shore movement and over what distances must the landing force be transported to the selected military objectives?

(4) What are the lift requirements of the separate tactical elements of the landing force?

A Marine Expeditionary Force (MEF), varying in size and composition according to the most probable conditions anticipated in each theater, is selected in this analysis as the landing force to be launched from amphibious assault ships of the Navy. The MEF is a flexible task organization which normally includes a Marine Division and a Marine Air Wing, together with other reinforcing elements. Different landing force requirements for varying contingencies are accommodated by adding or subtracting units from the MEF. For example, additional tank units can be added, or helicopter units subtracted, depending on the particular situation.

The major maneuver elements of the Marine Expeditionary Force are the Regimental Landing Teams (RLT) which vary for the same reasons in size and composition. A helicopterborne RLT is smaller, and substantially ligher than the surfaceborne RLT, for example. An MEF ordinarily contains three RLT's. The representative tactical plan for each of the four theaters is expressed in terms of the means by which these RLT's are to be landed and the probable conditions of distance and time which are the minimum acceptable to ensure an effective assault capability for the probable theater contingency, and accordingly represent maximum output mission requirements.

Table I below contains a summary of pertinent characteristics of each theater as described above and presents tactical aspects of the plans which reflect the differences identified in the operational environments and Marine landing force composition among the four theaters.[1]

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[1] There are, of course, an infinity of possible plans for each theater. Tactical plans are as varied as those who draft them. Those postulated in Table I are intended to highlight relative differences between theaters, and not probable absolute magnitudes.

TABLE I THEATER CHARACTERISTICS AND TACTICAL PLANS

THEATER OPERATIONAL CHARACTERISTICS					REPRESENTATIVE THEATER TACTICAL PLAN					
THEATER	DOMINANT TERRAIN CHARACTERISTICS	SEA/BEACH	EXPECTED ENEMY OPPOSITION		MEANS OF LANDING RLT'S	DISTANCE (miles)				TIME REQUIRED FOR INITIAL ASSAULT OPERATIONS.
						HCPTR		SURFACE		
			On Land	At Sea		Water	Land	Water	Land	
SEASIA	1. Jungle 2. Roads limited or nonexistent 3. Clearings for hcptr landings available	1. Adequate sea 2. Shoaling waters surrounding beach approaches 3. Adequate beaches	1. Limited on the beach 2. Primarily infantry 3. No tactical aviation	None	1. 2 RLT's by hcptr. 2. 1 RLT by armored amphib.	15	5	10	1	1. Hcptr - 1 hr. 2. Armored Amphib. - 1 hr. 3. Other surface - 4 hrs. 4. Complete Initial Ship-to-Shore -- 7 days
MIDEAST	1. Desert 2. Unlimited ground mobility 3. Unlimited Hcptr landing sites	1. Moderately restricted sea approaches 2. Adequate depths for beach approaches 3. Adequate beaches	1. Possible defended beaches 2. Primarily mobile armored defense 3. Limited tactical aviation	Limited shore-based tactical aviation	1. 2 RLT's by armored amphib. 2. 1 RLT by hcptr	20	10	15	2	1. Armored Amphib. - 1 hr. 2. Hcptr - 1.5 hrs. 3. Other surface - 3 hrs. 4. Complete Initial Ship-to-Shore -- 5 days
NOREUR	1. Cultivated/ industrial, mountainous 2. Well-developed road systems 3. Hcptr landing sites available in cultivated areas	1. Restricted sea approaches 2. Adequate depths for beach approaches 3. Limited beaches	1. Probable heavily defended beaches 2. Entrenched as well as mobile armored defense 3. Significant tactical aviation	Significant Naval forces & tactical aviation	1. 2 RLT's by armored amphib. 2. 2 RLT's by hcptr.	30	10	20	5	1. Armored Amphib. - 1 hr. 2. Hcptr - 2 hrs. 3. Other surface - 2.5 hrs. 4. Complete Initial Ship-to-Shore -- 3 days
COUNTER-INSURGENCY (CI)	1. All Types	1. Any within limits of navigation and beach crossing capability	1. Terrorism, sabotage, subversion 2. Guerrilla	None	1. 3 RLT's by hcptr.	5	45	5	10	1. Hcptr - 4.0 hrs. 2. Surface - 4.0 hrs.

The Tactical Planning Process and Determination of Output Mission Lift Requirements

Output mission lift requirements to be met by the amphibious force are extracted from the representative tactical plans of the four theater commanders (SEASIA, MIDEAST, NOREUR and CI) and are presented separately from other output mission parameters. This is done for the purpose of describing the planning process in general terms.

Tactical Planning

Tactical planning is done by each theater commander in a series of interdependent steps which are influenced by the mission assigned and the factors of expected enemy opposition, terrain, and beach and sea conditions described above.

First, military objectives are selected (terrain features, enemy forces) ashore which will provide for accomplishment of the overall landing force mission to seize the force beachhead. At this time, a general estimate is made of the overall size and type of landing force needed to accomplish the amphibious assault as well as subsequent operations. Next a scheme of maneuver is prepared which describes how and by what elements of the landing force these objectives will be attacked. The scheme of maneuver thus designates military objectives for the major maneuvering elements of the MEF and also directs at what time and how these elements will land: by helicopter, or armored amphibian vehicle. The evaluation of the size of the force needed is then refined and a task organization specified which allocates units to maneuver elements (RLT's) and other subordinate tasks.

Tactical planning decisions are now made as to how and when the remainder of the MEF is to be landed. This portion of the landing force is substantially larger than the RLT's and includes major heavy fire support and logistic and administrative elements. Part of this remainder must be landed by general amphibious vehicles (other than armored amphibians) -- heavy fire support units such as tanks and artillery are characteristic of this assault requirement. The rest of the landing force is generally landed by any means available (except armored amphibians which remain ashore) during a time period of 3-7 days.

To summarize the tactical planning sequence:

First -- military objectives to accomplish the overall mission of the landing force are selected.

Second -- a scheme of maneuver is prepared, the landing force task organized for the attack, and missions assigned to the task organized elements, including objectives, times and means of assault.

Third -- decisions are made as to how and when the remainder of the MEF will land.

The fourth and final step is to quantify the requirements generated by the planning sequence and assign output missions to the amphibious force for the execution of the ship-to-shore movement which is integrated with and directly supports the landing force scheme of maneuver and the related task organization. (Those aspects of the output mission imposed by the tactical plan which are related to time, distance, and means of assault are shown in Table I).

Determination of the Output Mission

Time and distance parameters are correlated with the physical size and weight of the men, equipment and supplies comprising the separate task elements to be delivered ashore. The magnitude of the total demand placed on the amphibious force is determined completely when the tonnages to be lifted in each output mission are identified. Landing force output mission tonnages can be seen to fall in four categories which are derived directly from the planning decisions of the commander.

1. Helicopterborne assault RLT's
2. Armored amphibious assault RLT's
3. Combat support forces not a part of the RLT's which are transported by general amphibious vehicle means
4. The remainder of the MEF which is unrestricted as to the type of surface vehicle required.[1]

Table II contains the lift requirements (in short tons) for each theater in each output mission and reflects the differently configured landing forces required in each instance. In the table, the tonnages for the separate output missions which identify the tasks of the amphibious assault vehicle force are listed for the SEASIA theater. A multiple is then assigned in each category for the other three theaters to reflect the differences in composition among the four forces. For example, for NOREUR the table shows the basic SEASIA MEF reinforced by one RLT (for a total of 4) and additional tank, and

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[1] This output mission is not considered by the analysis but is shown for its explanatory value in the description of the overall MEF lift requirement (see Footnote 3, Table II).

other heavy fire support units needed to initiate combat on the European mainland. Similarly, the reduced helicopter requirements for the MIDEAST scenario are reflected by a multiple of ½ of the SEASIA requirement.[1]

The output mission requirements, Table II, constitute the elements of the theater requirement vectors L_i, i = 1,...,4.

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[1] Estimates in Table II for weight requirements are generally based on unclassified planning factors contained in Marine Corps Schools planning exercize "Apex" conducted in May-June 1967. Although the estimates are believed generally representative, no pretense is held that they are accurate forecasts of actual future Marine landing force lift requirements.

TABLE II

OUTPUT MISSION LIFT REQUIREMENT[1]

MEF Output Mission Lift Requirement (short tons)	Theater Multiple			
	SEASIA (Vector L_1)	MIDEAST (Vector L_2)	NOREUR (Vector L_3)	CI (Vector L_4)
1. HCPTR[2] (Aslt Elms 2 RLT)				
a. Unlimited - 1900	1	½	1	1.5
b. Outsize - 100	1	½	1	1.5
2. Armored[2] Amphib (Aslt Elms 1 RLT)				
a. Unlimited - 1500	1	2	2	0
3. General[2] Amphib				
a. Unlimited - 6000	1	1.5	1.5	1
b. Outsize - 2000	1	2.0	2.0	1
4. Unrestricted[3]				
a. Unlimited - 54,000	1	1.1	1.3	.75
b. Outsize - 2000	1	2.0	2.0	0
TOTAL	60000	71000	82000	51500

[1] Outsize tonnages are the summed weights of individual lifts which exceed the capability of at least one of the candidate vehicles in any output mission. Unlimited tonnages can be embarked by any candidate vehicle operated in a particular output mission.

[2] Initial assault output mission lift requirements (incl. troops).

[3] This portion of the output mission is not included in the analysis. Earlier computer solutions of the linear program revealed that the initial assault output mission for general amphibious vehicles (which must be met before the unrestr. output mission) produced more vehicles than necessary for the unrestricted mission by several orders of magnitude.

APPENDIX 2

AMPHIBIOUS VEHICLE AND SHIP VECTOR DESCRIPTION

Resource Vectors

The analysis is directed toward solution of the vectors Q_1 (vehicles) and Q_2 (ships). Tables III and IV identify the composition of these vectors by individual vector element and describe the general characteristics of the corresponding candidate vehicles and ships. Capabilities of the vehicles and ships are listed in Tables VIII and IX, Appendix 4. Costs are tabulated in Table VI, Appendix 3.

TABLE III

CANDIDATE VEHICLES (Resource Vector Q_1)[1]

Vehicle	Vector Q_1 Element	Description
CH53	1	Cargo Helicopter - a heavy assault, single rotor, ramp loading helicopter capable of lifting all landing force equipment normally required for helicopterborne operations.
CH46	2	Cargo Helicopter - a medium assault tandem rotor, ramp loading helicopter capable of carrying the major part of landing force equipment used in helicopterborne operations.
LVTP	3	Landing Vehicle, Tracked, Personnel - an armored amphibian vehicle required for breaching established beach defenses and for subsequent mechanized operations.

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[1] The list of candidate vehicles could be expanded to include many more types. Those selected are considered most feasible (by the writer) because they are either: a) available now, b) proven through prototype test, or c) are extensions of proven vehicles.

TABLE III - Continued

Vehicle	Vehicle Q_1 Element	Description
LVTX	4	Landing Vehicle Tracked - A new armored amphibian vehicle, similar to the LVTP but somewhat smaller with lower payload offset by improved water and land speeds.
SK-10	5	A proposed air cushion vehicle with amphibious capabilities sufficient to embark all equipment of the landing force.
LCA	6	Landing Craft Assault - a new amphibious vehicle mounting a tank track and suspension system capable of embarking most landing force equipment except tanks.
LARC-60	7	Lighter, Amphibious, Resupply, Cargo - a large, wheeled amphibious vehicle capable of embarking all landing force equipment.
LCU	8	Landing Craft Utility - a large, non-amphibious landing craft which must discharge its payload or be off-loaded at the water's edge (capable of embarking all landing force equipment).
LCM-8	9	Landing Craft Medium - similar to the LCU, but smaller.

TABLE IV

CANDIDATE SHIPS (Resource Vector Q_2)[1]

Ship	Vector Q_2 Element	Description
LPH	1	Landing Platform, Helicopter (officially "Amphibious Assault Ship") - a large helicopter carrier with flight and hangar decks to embark and operate helicopters only.
LPD	2	Landing Platform, Dock - an amphibious ship configured with a flight deck and small well deck capable of operating all vehicles but not embarking helicopters.
LSD	3	Landing Ship, Dock - an amphibious ship with a large well deck capable of embarking and operating all vehicles except helicopters.
LST	4	Landing Ship, Tank - an amphibious ship with a large tank deck used to embark armored amphibians. (LST's are themselves capable of beaching but are not employed in that role in this analysis.

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[1] Certain older ship classes currently in use are not feasible candidates for embarking and operating the vehicle types considered by the analysis. These classes include APA's (Attack Personnel Transports) and AKA's (Attack Cargo Transports).

Theater Vehicle Employment Vectors

The elements of the theater resource employment vector X_i, $i=1,\ldots,4$ identify:

1. The output mission(s) in which the vehicle operates
2. The ship type(s) in which the vehicle is embarked to perform that mission.

This formulation permits a vehicle to operate from more than one ship type to accomplish the output mission(s) for which it is selected as a feasible candidate. Table V lists the elements of the vector X_i by ship and output mission.

As an example of the interpretation given theater employment vector elements, consider the SK-10. The SK-10 performs in two output missions, general amphibious unlimited and general amphibious outsize (see Table II, Appendix 1). Table V shows, however, that it can engage in these missions when operating from two different ship types - the LPD (elements 6 and 7) or the LSD (elements 17 and 18).

The effect of the technique on the analysis is significant. Individual ship and vehicle types do not compete directly for selection, but in combinations of ships and vehicles. To see that the method makes intuitive sense, consider again the elements 6 and 17. These elements are respectively the number of SK-10's operated from LPD's and LSD's. Since the two ship types exhibit different costs and capacities, it seems reasonable that one combination may be preferred to the other.

Perhaps the best way to grasp the meaning of the 27 elements of the employment vector is to reason that each vehicle would perform its output mission(s) in different ways if embarked in different ship types and hence each of the elements represents a separate technique for delivering Marine landing force units from ship to shore.

TABLE V

THEATER VEHICLE EMPLOYMENT VECTORS X_i, $i=1,\ldots,4$[1]

Vehicle	Class Ship In Which Embarked	Output Mission[2]	Vector X_i Element
CH 53	LPH	HCPTR	1
CH 53	"	" (O)	2
CH 46	"	"	3
LVTP	LPD	ARMORED AMPHIB	4
LVTX	"	"	5
SK-10	"	GEN AMPHIB	6
SK-10	"	" (O)	7
LCA	"	"	8
LARC-60	"	"	9
LARC-60	"	" (O)	10
LCU	"	"	11
LCU	"	" (O)	12
LCM-8	"	"	13
LCM-8	"	" (O)	14
LVTP	LSD	ARMORED AMPHIB	15
LVTX	"	"	16
SK-10	"	GEN AMPHIB	17
SK-10	"	" (O)	18
LCA	"	"	19
LARC-60	"	"	20
LARC-60	"	" (O)	21
LCU	"	"	22
LCU	"	" (O)	23
LCM-8	"	"	24
LCM-8	"	" (O)	25
LVTP	LST	ARMORED AMPHIB	26
LVTX	"	"	27

---

[1] Output missions are described in Appendix 1, p. 69.

[2] (O) denotes that the vehicle operates in both outsize and unlimited output mission sub categories.

The Distribution Submatrix K

The theater vehicle employment vector X_i contains 27 elements which identify particular levels of 9 candidate vehicles embarked in 4 ship classes competing in 5 output missions. The mathematical device of the central and theater primal linear programs which is used to correlate this vector with the (total) vehicle vector Q_1 in the constraint matrix is the submatrix K. The distribution submatrix K is a matrix of unit coefficients which, when pre-multiplied times the employment vector X_i results in summation of the levels of each vehicle type operating in designated output missions distributed over all ship classes for comparison with the 9 resource elements of the vector Q_1.

APPENDIX 3

COSTS

Costs considered in the analysis are of two types: Systems costs, which include investment and operating costs associated with peacetime procurement and operation of the amphibious force, and expected wartime attrition costs.

For purposes of the analysis, it is assumed that the entire existing amphibious vehicle force must be replaced and that there are no inherited assets from the present force. It is further assumed that there is no salvage value at the end of the planning cycle and that all vehicles and ships have a ten-year system life. Costs of candidate amphibious ships programmed to be available before 1970 are treated as sunk.

The planning cycle selected is the 10-year period 1970-79. All of the vehicles considered (Appendix 2) are either available now or can be in production in sufficient time to achieve a readiness date in 1970. No development costs are attributed to any of the vehicles since development work on even the more advanced types (SK-10, LCA) is substantially complete and acquisition costs are constant over any procurement range.

Investment and Peacetime Operating Costs

Total systems costs of investment and peacetime operation are computed as of the year 1970. That is, all investment is assumed to take place in that year and is not discounted. Peacetime operating costs are discounted at 10% over the period. These costs are shown in Table VI.

Since the problem is one of determining the relative cost-effectiveness of the amphibious vehicles and ships, the preceding costing assumptions do not favor any particular vehicle. This is true in particular of those vehicles available in the future for which substantially higher relative performance is anticipated. Investment costs, if discounted in terms of time of availability, could weight the analysis in the direction of these more productive vehicles because of the apparent (when viewed from the present) lower costs. Investment and discounted peacetime operating costs of the vehicles and ships are respectively the elements of the cost vectors C_1 and C_2 associated with the resource vectors Q_1 and Q_2.

Expected Attrition Costs

The inclusion of expected attrition costs in the analysis is at once the most sensitive aspect of the costing procedure and the most interesting. In the analysis, expected attrition costs are determined for each theater (SEASIA, MIDEAST, NOREUR, CI) as a function of the estimated probability of a requirement to conduct one amphibious operation in the particular theater, the expected total attrition for each vehicle during the assumed period of the initial assault and the (undiscounted) replacement cost.

Let N - probability of conducting an amphibious operation in a particular theater

A - expected attrition for each vehicle in each theater

R - replacement cost (equal to initial investment cost)

P - expected attrition costs for each vehicle type in each theater

Then $P = N A R$

That is, the expected attrition cost in the ith theater is the product of the probability of one amphibious operation in the ith theater, expected vehicular attrition, and replacement cost. The expected attrition costs computed for each vehicle in each theater become the elements of the expected attrition cost vector, P_i, i= 1,...,4.[1]

Table VI lists the values of these cost parameters. Clearly, the entire analysis is sensitive to the magnitude of attrition A and the probability of an amphibious operation N. The asserted values are estimates based entirely on the writer's judgment. These parameters are themselves the subject of much analysis, none of it conclusive and all of it classified. Nothing is claimed for them except that they are believed to be at least a reasonable estimate of the relative ordinal ranking of probabilities between theaters and expected attrition among the various vehicle types and provide a basic input needed for the analysis. It is only suggested that the model constructed for the present analysis of the problem of force selection is general in nature and will accommodate any value which may be preferred to those chosen.

Landing Craft Costs

A special case of the general costing procedure which requires amplification is that of the non-amphibious landing craft (the LCU and

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[1] No attrition is assumed for amphibious ships, hence no expected attrition costs are attributed to ships.

TABLE VI COSTS ($X10^6$)

Vehicle or Ship	Vectors Q_1, C_1 Q_2, C_2 Element	10 YR. SYSTEM COST/VEHICLE OR SHIP					THEATER EXPECTED ATTRITION COSTS/VEHICLE[1]													
		Invest	Undisc. 10 yr. Oper.	Disc. (10%) 10 yr. Oper.	Total Undisc. Invest. & Oper.	Vectors C_1, C_2 Total Invest. & Disc. Oper,	Attrition per Vehicle				Prob. of Operation				Replace/ Vehicle	Expected Attrition Cost/Vehicle P_i				Vector P_i Element
							SEA	NE	ME	CI	SEA	NE	ME	CI		SEA	NE	ME	CI	
Vehicle[2,4]	Q_1, C_1					C_1														
CH 53	1	2.0	3.6	2.2	5.6	4.2	.1	.3	.2	.05	.5	.05	.2	.8	2.0	.1	.030	.08	.08	1,2
CH 46	2	1.0	3.0	1.835	4.0	2.835	.1	.3	.2	.05		-do-			1.0	.05	.015	.04	.04	3
LVTP	3	.13	.406	.248	.536	.378	.1	.4	.3	N/A		-do-			.13	.0065	.0026	.0078	N/A	4/15/26
LVTX	4	.065	.341	.208	.406	.273	.09	.39	.29	N/A		-do-			.065	.0029	.0013	.0039	N/A	5/16/27
SK-10	5	3.6	3.968	2.43	7.568	6.03	.05	.2	.08	.01		-do-			3.6	.09	.036	.0576	.0288	6,7/17,18
LCA	6	.3	.576	.352	.876	.652	.15	.35	.2	.01		-do-			.3	.0225	.0053	.012	.0024	8/19
LARC-60	7	.35	.626	.383	.976	.733	.15	.4	.25	.02		-do-			.35	.0263	.007	.0175	.0056	9,10/20,21
LCU	8	2.09	8.47	5.195	10.56	7.285	.1	.2	.15	.01		-do-			2.090	.1045	.0209	.0627	.0175	11,12/22/23
LCM-8	9	.477	2.34	1.428	2.817	1.905	.11	.21	.16	.02		-do-			.477	.0262	.005	.015	.0076	13,14/24,25
Ships[3,5]	Q_2, C_2					C_2														
LPH	1	40	64.47	39.31	104.47	79.31														
LPD	2	41	63.66	38.82	104.66	79.82														
LSD	3	36	49.93	30.45	85.93	66.45														
LST	4	25	37.70	21.77	46.77	46.77														

Footnotes

[1] See Table V, Appendix 1 for correlation of the P_i vectors with output missions, ship classes and theater employment vectors X_i, i = 1,...,4 . Element numbers designate ships in which embarked by output mission as follows: LPH: 1-3; LPD: 4-14; LSD:15-25; LST:26,27

[2] Vehicle costs are extracted directly or extrapolated from a Food Machinery Corporation Study, Mission Effectiveness Analysis for Cargo Transfer Vehicles (undated).

[3] Ship Costs are extracted from Jane's Fighting Ships.

[4] Annual operating costs are determined using the standard Navy procedure: 10% hull cost + crew x \$4600 = annual operating cost.

[5] Ships available by 1970 (according to Jane's Fighting Ships) whose investment costs are treated as sunk are:

LPH-	7
LPD-	16
LSD-	5
LST-	18

the LCM-8). In order to compare the effectiveness of these craft with other candidate vehicles it is necessary to develop a supporting system of equipment which will permit the landing craft to compete for a role in the amphibious ship-to-shore mission. It is first of all apparent that these craft cannot themselves deliver tonnages inland. The payload of landing craft must be off-loaded at the shore-line and subsequently transported inland the distance specified by the tactical plan.

In present-day amphibious operations the capability to off-load landing craft at the beach is furnished by the shore party battalion which includes the necessary cargo handling equipment. The means to transport the landing force materiel delivered by landing craft to inland locations is provided by motor transport battalions. It is emphasized that these organizations are not required to support the operations of the other vehicles of the candidate force, except for a small command and control element of the shore party battalion which would be required to control landward activities of the ship-to-shore movement. Total costs of landing craft in Table VI reflect the following contributory costs which are in addition to ordinary LCU and LCM-8 system costs of investment and peacetime operation.

1. Three-fourths of the investment and peacetime operating costs of a shore party battalion allocated 1/3 to the LCM-8 and 2/3 to the LCU based on estimated numbers of LCU's and LCM-8's required to support present-day MEF level operations.

2. The cost of procuring and operating that number of 5-ton trucks needed to transport the payload of each landing craft (40 trucks to support each LCU and 12 trucks for each LCM-8).

Table VII lists the costs developed to reflect the "true" costs of landing craft for comparison with amphibious vehicles.[1]

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[1] The cost penalty assessed against the LCU and LCM-8 for their non-amphibious characteristics is somewhat arbitrary and other allocations could obviously be used. Nevertheless, some penalty is required. Otherwise, these craft (which are the backbone of the present-day force) could not compete at all!

TABLE VII

LANDING CRAFT COSTS ($X10^6$) [1]

Cost Category	LCU	LCM-8
Landing Craft		
Invest	1.3	.2
10-yr. Operate (discounted)	1.84	.37
Shore Party		
Invest	.054	.027
10-yr. Operate (discounted)	.255	.128
Motor Transport		
Invest	.736	.22
10-yr. Operate (discounted)	3.1	.93
TOTAL	7.285	1.905

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[1] Shore party and motor transport costs are investment + 10 yr. discounted operating costs for these units obtained from appropriate sections in Headquarters, U.S. Marine Corps.

APPENDIX 4

VEHICLE OUTPUT COEFFICIENT AND SHIP CAPACITY

The constraints of the basic primal linear program from which the decomposable dual is derived are related to specific amphibious force output missions for delivery of the landing force ashore. These output missions are expressed in terms of the tonnages, times available and distances specified for the particular ship-to-shore movement (see Appendix 1).

Vehicle Output Coefficient

The primary measure of effectiveness selected for evaluation of the vehicle force is the relative capability of the various combinations of ships and vehicles to transport and land the landing force tonnages (output missions). Thus the magnitude and distribution of landing force combat power ashore delivered to particular objectives in a specified period of time are considered adequate measures of effectiveness for the selection of the amphibious force.

The aggregate effectiveness of a particular combination of vehicles in an output mission depends upon the separate lift contribution of each vehicle in the selected force. Consequently an "output" coefficient has been derived which permits computation of the relative productivity of each vehicle.

These values are then related to each amphibious vehicle vector X_i, $i=1,\dots,4$ in the constraint matrix, showing its contribution toward the tons delivered output mission. The

coefficient is computed by determining the number of round trips which the vehicle can complete in a given time period multiplied by its capacity in tons (including fractions of round trips above .5, i.e. the candidate vehicle must at least be capable of reaching the beach once in the time allowed).

For each vehicle in each scenario let:[1]

E = Output coefficient (tons)

T = Time permitted for the initial surface assault, or helicopter assault (hours)

D_W = Overwater distance (miles)

D_L = Overland distance (miles)

V_W = Vehicle water speed (mph)

V_L = Vehicle land speed (mph)

L = Vehicle loading and unloading time (hours)

C = Vehicle capacity (short tons)

Then: $$E = C \left[\frac{T}{\dfrac{2D_W}{V_W} \qquad \dfrac{2D_L}{V_L} + L} \right]$$

The computational formula expresses the tonnage delivered by the individual vehicle in the time and over the distance specified by the given theater tactical plan where the expression

$\dfrac{2D_W}{V_W} + \dfrac{2D_L}{V_L}$ is the time required for one round trip.

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[1] Distances and speeds in nautical miles.

The values E are coefficients of the X_i vectors in each theater, distributed over the ships by output mission in which each vehicle competes and thus form the elements of the matrices E_i, i=1,...,4.

Table VIII contains values of E computed using the formula above. The table also contains significant input and intermediate values used in the computation. Times and distances used are extracted from the tactical plan for each theater contained in Table I, Appendix 1.

The Output Coefficient Submatrix E_i, i=1,...,4.

The submatrix E_i is composed of unit output coefficients of the amphibious vehicle force. It functions in the central and theater primal linear program constraint matrices to correlate the aggregate output of the force in a particular theater with the theater output mission vector L_i, i=1,...,4. The submatrix E_i when pre-multiplied times the theater employment vector X_i sums the output of all vehicle types over all ship types by output mission. One element of E_iX_i, for example, adds the output tonnages of the SK-10's, LCA's, LARC-60's, LCU's and LCM-8's embarked in both LPD's and LSD's engaged in meeting the general amphibious unlimited output mission in an individual theater.

Ship Embarkation Capacity

Table IX tabulates ship embarkation capacity for each vehicle type. The inverse of the particular ship's capacity for a certain type of vehicle is the per unit requirement of the vehicle for that ship class. These inverse values are the elements of the per unit vehicle ship embarkation capacity requirement matrix F.

TABLE VIII VEHICLE OUTPUT COEFFICIENTS

VEHICLE			VEHICLE PERFORMANCE[4]				THEATER DELIVERY CAPABILITY															
TYPE	OUTPUT MISSION	CORRELATED ELEMENT OF VECTOR X_i[3]	PAYLOAD		SPEED (mph)		ROUND TRIP TIME (Hrs)								NUMBER OF ROUND TRIPS[5]				OUTPUT COEFFICIENT E (short tons)			
			Load/ Unload Time (hrs)	Weight (short tons)	Water	Land	Water[1]				Land[2]											
							SEA	ME	NE	CI	SEA	ME	NE	CI	SEA	ME	NE	CI	SEA	ME	NE	CI
CH-53	Hcptr	1,2	.4	4	150	150	.28	.4	.53	.66					4.41	1.88	1.08	3.78	17.2	7.52	4.30	15.1
CH-46	Hcptr	3	.4	2	130	130	.31	.46	.615	.77					4.22	1.74	.985	3.41	8.45	3.48	1.97	6.82
LVTP	Armored Amphib.	4/15/26	.05	7	6	20	.67	.67	.67	N/A	.05	.1	.25		1.3	1.22	1.03	N/A	7	7	7	N/A
LVTX	Armored Amphib.	5/16/27	.05	5	8	25	.5	.5	.5	N/A	.04	.08	.2		1.7	1.59	1.33	N/A	5	5	5	N/A
SK-10	Gen. Amphib	6,7/17,18	.4	60	60	25	.33	.5	.66	.167	.04	.14	.4	.4	5.2	2.88	1.7	4.15	312	173.1	102.8	294
LCA	Gen. Amphib	8/19	.3	30	12	18	1.66	2.5	3.33	.833	.055	.22	.56	.556	1.97	.99	.596	2.37	59.2	29.7	17.9	71
LARC-60	Gen. Amphib	9,10/20,21	.3	60	6	14	3.33	5	6.67	1.67	.071	.29	.71	.715	1.08	.537	.325	1.49	64.8	32.2	0	89
LCU	Gen. Amphib	11,12/22,23	.8	200	11	25	1.82	2.73	3.6	.91	.04	.14	.4	.4	1.5	.817	.52	1.9	300	163.5	104	380
LCM-8	Gen. Amphib	13,14/24,25	.6	60	10	25	2	3	4	1	.04	.14	.4	.4	1.5	.80	.5	2	91	48.13	30	120

Footnotes

[1] a. The total helicopter round trip time is listed in the water columns.
b. Water distance for LVT's is 4000 yards for all theaters under the assumption that ships embarking these vehicles close the beach to that range.

[2] LVT's are not considered as required in the ship-to-shore assault role for the CI scenario.

[3] Vector X_i elements: LPH: 1-3 LPD: 4-14 LSD: 15-25 LST: 26-27

[4] Performance information is extracted directly or extrapolated from a Food Machinery Corporation Study, Mission Effectiveness Analysis for Cargo Transfer Vehicles (undated), FM 101-10, Staff Officers Field Manual, Logistic, Technical and Logistic Reference Data, and Bell SK-10 Air Cushion Vehicle, Bell Aerosystems Report D7268-953001, May 1966.

[5] Time and distance parameters used in computing the number of round trips are extracted from Table I, Appendix 1.

TABLE IX

SHIP EMBARKATION CAPACITY[1,2]

Ships → Vehicles ↓	LPH 1-3	LPD 4-14	LSD 15-25	LST 26-27	← Vector X_i[3] Elements ↓
CH53	24	0	0	0	1,2
CH46	36	0	0	0	3
LVTP	0	43	54	22	4/15/26
LVTX	0	66	83	33	5/16/27
SK-10	0	2	4	0	6,7/17,18
LCA	0	4	9	0	8/19
LARC-60	0	4	9	0	9,10/20,21
LCU	0	1	3	0	11,12/22,23
LCM-8	0	4	9	0	13,14/24,25

[1] These embarkation capacities are very rough estimates based generally on unclassified ship and vehicle characteristics contained in the Amphibious Planning Exercise APEX previously cited.

[2] A zero entry in the table indicates that the ship is not suitable for embarking and operating the particular vehicle type.

[3] See Table V, Appendix 1 for correlation with theater output missions employment vectors X_i, $i=1,\ldots,4$.

The Ship Vehicle Embarkation Requirement Submatrix F

Elements of the ship vector Q_2 are the levels of amphibious ships needed to embark the vehicle force. The F submatrix of the constraint matrices in the central and theater primal linear programs provides for comparison of theater requirements for amphibious ships with the vector Q_2. The elements of the F submatrix are the per unit vehicle requirements for amphibious ships. For example, elements of F correlated with X_i elements 4, 15 and 26 have the values 1/43, 1/54 and 1/22. These are respectively that portion of the total LVTP capacity of an LPD, LSD and LST needed to embark one LVTP.

Premultiplying the theater employment vector X_i by the submatrix F results in the summation of vehicle requirements for amphibious ships by ship class. One element of the submatrix FX_i adds, for example, the number of LPD's embarking SK-10's in the general amphibious unlimited role to the number of LPD's embarking the LVTX operating in the armored amphibian output mission and so on, until total theater usage of LPD's is determined.

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